

SNS COLLEGE OF TECHNOLOGY



(An Autonomous Institution) Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT) COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT II

ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

Quadratic_form ; A homogeneous polynomial of the Second degree in any number of variables is called Quadratic form. Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ $X = \begin{pmatrix} \varkappa_{1} \\ \varkappa_{2} \\ \varkappa_{3} \end{pmatrix} \quad \& \quad X_{1}^{T} = \begin{pmatrix} \varkappa_{1} & \varkappa_{2} & \varkappa_{3} \end{pmatrix}$ $Q = X^T A X$ $Q = (\chi_1, \chi_2, \chi_3) \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$ $Q = a_{11} x_{1}^{2} + a_{22} x_{2}^{2} + a_{33} x_{3}^{2} + 2a_{12} x_{1} x_{2}$ $+ 2a_{23} x_2 x_3 + 2a_{31} x_3 x_1 \longrightarrow (1)$ Here $a_{21} = a_{12}$, $a_{31} = a_{13}$, $a_{23} = a_{32}$ Equation () is called the matrix of the Quadratic form. Note : $Q = \begin{bmatrix} \operatorname{Coef} & \operatorname{of} & \chi^2 & \frac{1}{2} \operatorname{coef} & \operatorname{of} & \chi y & \frac{1}{2} \operatorname{Coef} & \chi z \\ \frac{1}{2} \operatorname{coef} & \operatorname{of} & y z & \operatorname{Coef} & \operatorname{of} & y^2 & \frac{1}{2} \operatorname{coef} & \operatorname{of} & y z \\ \frac{1}{2} \operatorname{coef} & \operatorname{of} & z \chi & \frac{1}{2} \operatorname{coef} & \operatorname{of} & z \end{pmatrix}$



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Nature of the quadratic form: Let $Q = X^T A X$, be a quadratic form in n vagiables x_1, x_2, \dots, x_n (i) Rank : Number of non-zero eigen values (ii) Index : Number of positive square teams in the canonical form. (iii) Signature : Difference between the number of Positive and negative savuares terms in the canonical form. (iv) Nature : Positive Definite : If all the eigen values are positive Positive Semi definite : If all the eigen values are positive and atleast one eigen value is zero. Negative Definite : If all the eigen values are negative. Negative Semi definite : If all the eigen values are negative and atleast one eigen value is Zero. Indefinite : If it has both positive and negative eigen values



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Problems : Find the matrix of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$ $\mathcal{Q} = \begin{bmatrix} \cos \beta & o \beta & x^2 & \frac{1}{2} \cos \beta & o \beta & xy & \frac{1}{2} \cos \beta & o \beta & xz \\ \frac{1}{2} \cos \beta & o \beta & yx & \cos \beta & o \beta & y^2 & \frac{1}{2} \cos \beta & o \beta & yz \\ \frac{1}{2} \cos \beta & o \beta & zx & \frac{1}{2} \cos \beta & o \beta & zy & \cos \beta & o \beta & z^2 \end{bmatrix}$ $=\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Write the matrix of the quadratic form. (2) (i) $x^2 + 2y^2$ 1 Single States $-\frac{1}{2} \circ \left(\begin{array}{c} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{array} \right)$ -4xy - 2yz + 4xz $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ support that the addition of