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**DEPARTMENT OF MATHEMATICS** 

#### UNIT III DIFFERENTIAL CALCULUS

Centre of Curvature.  $\overline{\chi} = \chi - \frac{g_1}{g_2} \left( 1 + g_1^2 \right)$  $\bar{y}_{1} = y + \frac{1}{y_{2}} (1 + y_{1}^{2})$ (x, y) is the Coordinate of the centre of curvature. Circle of Curvature : The equation of the circle of cuavature is  $(x-\overline{x})^{2} + (y-\overline{y})^{2} = P^{2}$ Problems :  $\bigcirc$  Find the centre of curvature. of  $y = x^2$  at the origin. soln:  $y = x^2$  Point: (0,0)  $\frac{dy}{dx} = y_1 = 2x ; y_1(0,0) = 2(0) = 0$  $\frac{d^2y}{dx^2} = y_2 = 2 ; \quad y_2(0,0) = 2$ The centre of curvature is,  $\overline{x} = x - \frac{y_1}{y_2} (1 + y_1^2) \int \overline{y} = y + \frac{1}{y_2} (1 + y_1^2)$ 





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3 Find the equation of the circle of curvature of the sectangular hyperbola xy = 12 at the Point (3,4) Soln : The equation of which of wature is given by,  $(x - \overline{x})^2 + (y - \overline{y})^2 = P^2 \longrightarrow (1)$ where.  $\overline{\chi} = \chi - \frac{y_1}{y_2} (1 + y_1^2)$  $\bar{y} = y + \frac{1}{y_2} (1 + y_1^2)$  $P = (1+y_{1}^{2})^{3/2}$ Griven: xy = 12 point: (3,4) $x \frac{dy}{dx} + y(1) = 0$  $\begin{array}{l} x \ y_{1} + y = 0 \\ x \ y_{1} = -y \end{array}$  $y_{1} = -\frac{y}{2}$   $y_{1}(3,4) = -\frac{4}{3}$  $\begin{array}{l} y_{2} = \frac{x (-y_{1}) - (-y)(1)}{x^{2}} \\ = \frac{-xy_{1} + y}{x^{2}} \end{array}$ 



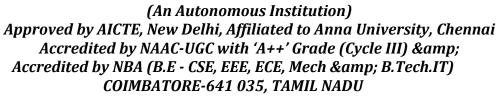


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$$\begin{aligned} \mathcal{Y}_{z}(3; \psi) &= \frac{-\overline{s}'\left(-\frac{\psi}{s'}\right) + \psi}{s^{2}} \\ &= \frac{4 + \psi}{q} \\ \boxed{\left[\overline{y_{z}} = \frac{8}{9}\right]} \\ \overline{x} &= x - \frac{\overline{y}_{1}}{y_{z}}(1 + y_{1}^{z}) \\ &= \overline{s} - \frac{\left(-\frac{\psi}{s}\right)}{\frac{8}{9}}\left(1 + \left(-\frac{\psi}{s}\right)^{2}\right) \\ &= \overline{s} + \frac{4}{s^{2}} \times \frac{9^{2}}{s_{z}}\left(1 + \frac{16}{9}\right) \\ &= \overline{s} + \frac{3}{2^{2}} \times \frac{9^{2}}{s_{z}}\left(1 + \frac{16}{9}\right) \\ &= \overline{s} + \frac{3}{2^{2}} \times \frac{25}{8^{3}} \\ &= \overline{s} + \frac{25}{6} \\ &= \frac{18 + 25}{6} \\ \boxed{\overline{x} = \frac{43}{6}} \end{aligned}$$







**DEPARTMENT OF MATHEMATICS** 

$$\begin{split} \overline{y} &= y + \frac{1}{y_2} \left( 1 + y_1^2 \right) \\ &= 4 + \frac{1}{\frac{8}{9}} \left[ 1 + \left( -\frac{4}{3} \right)^2 \right] \\ &= 4 + \frac{9}{8} \left[ 1 + \frac{14}{9} \right] \\ &= 4 + \frac{9}{8} \left( \frac{9 + 14}{9} \right) \\ &= 4 + \frac{9}{8} \left( \frac{9 + 14}{9} \right) \\ &= 4 + \frac{9}{8} \left( \frac{9 + 14}{9} \right) \\ &= 4 + \frac{9}{8} \left( \frac{25}{8} \right)^{3/2} \\ &= \frac{1 + \frac{9}{8} \left( \frac{9 + 14}{9} \right)^2}{\frac{3}{2}} \\ &= \frac{\left[ 1 + \left( -\frac{4}{3} \right)^2 \right]^{3/2}}{\frac{8}{9}} \\ &= \frac{\left[ 1 + \left( -\frac{4}{3} \right)^2 \right]^{3/2}}{\frac{8}{9}} \\ &= \frac{\left[ 1 + \left( -\frac{4}{3} \right)^2 \right]^{3/2}}{\frac{8}{9}} \\ &= \frac{\left[ \frac{5}{3} \left( \frac{25}{9} \right)^{3/2}}{\frac{8}{9}} \\ &= \frac{\left[ \frac{5}{3} \left( \frac{25}{3} \right)^{3/2} \right]^2}{\frac{8}{9}} \\ &= \frac{125}{\frac{8}{3}} \times \frac{9}{8} \\ &= \frac{125}{\frac{27}{3}} \times \frac{9}{8} \\ &= \frac{125}{\frac{125}{3}} \times \frac{9}{8} \\ &= \frac{125}{\frac{125}{3}} \\ &= \frac{125}{3} \\ &= \frac{125}{3}$$

23MAT101 - MATRICES AND CALCULUS





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 $\left(\frac{125}{2^{"}}\right)$  $= \left(\chi - \frac{43}{6}\right)^2 + \left(\frac{y - 57}{8}\right)^2$ -