



DEPARTMENT OF MATHEMATICS

UNIT III

DIFFERENTIAL CALCULUS

Centre of curvature

$$\bar{x} = x - \frac{y_1 (1 + y_1^2)}{y_2}$$

$$\bar{y} = y + \frac{1 (1 + y_1^2)}{y_2}$$

(\bar{x}, \bar{y}) is the coordinate of the centre of curvature.

Circle of curvature:

The equation of the circle of curvature is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$$

Problems:

- ① Find the centre of curvature of $y = x^2$ at the origin.

Soln:

$$y = x^2$$

Point: $(0, 0)$

$$\frac{dy}{dx} = y_1 = 2x \quad ; \quad y_1(0,0) = 2(0) = 0$$

$$\frac{d^2y}{dx^2} = y_2 = 2 \quad ; \quad y_2(0,0) = 2$$

The centre of curvature is,

$$\bar{x} = x - \frac{y_1 (1 + y_1^2)}{y_2}$$

$$\bar{y} = y + \frac{1 (1 + y_1^2)}{y_2}$$

$$= 0 - 0(1 + 0)$$

$$= 0 + \frac{1}{2}(1 + 0)$$

$$\bar{x} = 0$$

$$\bar{y} = \frac{1}{2}$$

$$\therefore (\bar{x}, \bar{y}) = (0, \frac{1}{2})$$



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- ② Find the equation of the circle of curvature of the rectangular hyperbola $xy = 12$ at the point $(3, 4)$.

Soln:

The equation of circle of curvature is given by,

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2 \rightarrow \textcircled{1}$$

where,

$$\bar{x} = x - \frac{y_1 (1 + y_1^2)}{y_2}$$

$$\bar{y} = y + \frac{1 (1 + y_1^2)}{y_2}$$

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$$

Given: $xy = 12$ point: $(3, 4)$

$$x \frac{dy}{dx} + y(1) = 0$$

$$x y_1 + y = 0$$

$$x y_1 = -y$$

$$y_1 = \frac{-y}{x}$$

$$y_1(3, 4) = \frac{-4}{3}$$

$$y_2 = \frac{x(-y_1) - (-y)(1)}{x^2}$$

$$= \frac{-xy_1 + y}{x^2}$$



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$$y_2(3,4) = \frac{-\cancel{3} \left(\frac{-4}{\cancel{3}} \right) + 4}{3^2}$$
$$= \frac{4+4}{9}$$

$$\boxed{y_2 = \frac{8}{9}}$$

$$\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2}$$
$$= 3 - \frac{\left(\frac{-4}{3} \right)}{\frac{8}{9}} \left(1 + \left(\frac{-4}{3} \right)^2 \right)$$

$$= 3 + \frac{4}{3} \times \frac{9^3}{8^2} \left(1 + \frac{16}{9} \right)$$

$$= 3 + \frac{3}{2} \left(\frac{9+16}{9} \right)$$

$$= 3 + \frac{3}{2} \times \frac{25}{9}$$

$$= 3 + \frac{25}{6}$$

$$= \frac{18+25}{6}$$

$$\boxed{\bar{x} = \frac{43}{6}}$$



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$$\begin{aligned}\bar{y} &= y + \frac{1}{y_2} (1 + y_1^2) \\ &= 4 + \frac{1}{\frac{8}{9}} \left[1 + \left(\frac{-4}{3} \right)^2 \right] \\ &= 4 + \frac{9}{8} \left[1 + \frac{16}{9} \right] \\ &= 4 + \frac{9}{8} \left(\frac{9+16}{9} \right) \\ &= 4 + \frac{9}{8} \times \frac{25}{9} = 4 + \frac{25}{8} = \frac{32+25}{8}\end{aligned}$$

$$\boxed{\bar{y} = \frac{57}{8}}$$

$$\begin{aligned}P &= \frac{(1 + y_1^2)^{3/2}}{y_2} \\ &= \frac{\left[1 + \left(\frac{-4}{3} \right)^2 \right]^{3/2}}{8/9} \\ &= \frac{\left[1 + \frac{16}{9} \right]^{3/2}}{8/9} = \frac{\left(\frac{25}{9} \right)^{3/2}}{8/9} \\ &= \frac{\left[\frac{5^2}{3^2} \right]^{3/2}}{8} = \frac{5^3}{3^3} \times \frac{9}{8}\end{aligned}$$

$$= \frac{125}{27} \times \frac{9}{8} = \frac{125}{24} \Rightarrow \boxed{P = \frac{125}{24}}$$



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$$\therefore \textcircled{1} \Rightarrow \left(x - \frac{43}{6}\right)^2 + \left(y - \frac{57}{8}\right)^2 = \left(\frac{125}{24}\right)^2$$