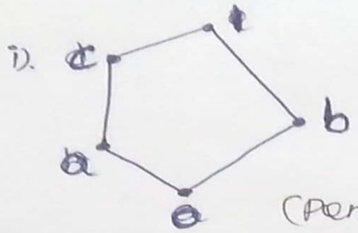
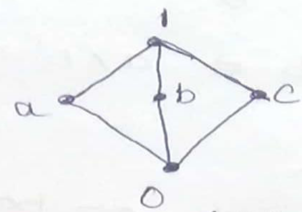
Determine which of the following lattices are modular.

i).



(Pentagon)

ii).



(Diamond)

i). consider (a, b, c)

clearly $a \le c$

Now LHS $= a \vee (b \wedge c)$

$= a \vee a$

$= a$

RHS $= (a \vee b) \wedge c$

$= c \wedge c$

$= c$

$a \neq c$

If $a \le c$, then $a \vee (b \wedge c) \neq (a \vee b) \wedge c$

M, condition is not satisfied.

N_5) Pentagon lattice is not a modular lattice.

ii). Diamond lattice is modular.

Theorem:

Every distributive lattice is modular but not conversely.

Proof.

Let $(L, \wedge, \vee)$ be the given distributive lattice.

D_1: $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, $\forall a, b, c \in L$ $\rightarrow (1)$

If $a \le c$ then $a \vee c = c \longrightarrow (2)$

$(1) \Rightarrow$ $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$= (a \vee b) \wedge c$ using (2)

∴ If $a \leq c$ then $(a \vee b) \wedge c = a \vee (b \wedge c)$

∴ Every distributive lattice is modular.

But, converse is not true.

ie, Every modular lattice need not be distributive.

For eg., M_5 (Diamond lattice) is modular but it is not distributive.