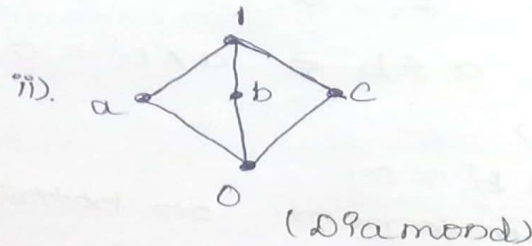
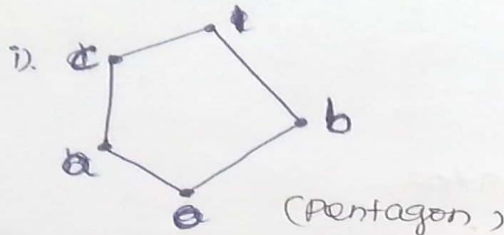




## DEPARTMENT OF MATHEMATICS

Determine which of the following lattices are modular.



ii) Consider  $(a, b, c)$

clearly  $a \leq c$

$$\begin{aligned} \text{Now LHS} &= a \vee (b \wedge c) \\ &= a \vee \underline{a} \\ &= a \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (a \vee b) \wedge c \\ &= \underline{c} \wedge c \\ &= c \end{aligned}$$

$$a \neq c$$

If  $a \leq c$ , then  $a \vee (b \wedge c) \neq (a \vee b) \wedge c$

∴ condition is not satisfied.

N<sub>5</sub>) Pentagon lattice is not a modular lattice.

ii). Diamond lattice is modular.

Theorem:

Every distributive lattice is modular but not conversely.

Proof

Let  $(L, \wedge, \vee)$  be the given distributive lattice.

$$D_1: a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c), \quad \forall a, b, c \in L$$

If  $a \leq c$  then  $a \vee c = c \xrightarrow{(1)}$

$$(1) \Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$



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## DEPARTMENT OF MATHEMATICS

$$= (a \vee b) \wedge c \quad \text{using (2)}$$

$\therefore$  If  $a \leq c$  then  $(a \vee b) \wedge c = a \vee (b \wedge c)$

$\therefore$  Every distributive lattice is modular.

But, converse is not true.  $\checkmark$

i.e., Every modular lattice need not be distributive.

For eg.,  $M_5$  (Diamond lattice) is modular but it is not distributive.