UNIT 5 Z - Transforms and Difference equations
Inverse Z – transforms using Partial Fractions

Inverse z-transform by using Partial fraction Method.

1. Find the inverse Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$

Let $F(z) = \frac{z^2+z}{(z-1)(z^2+1)} = \frac{Az}{z-1} + \frac{Bz^2+Cz}{z^2+1}$

$z^2+z = Az(z^2+1) + (Bz^2+Cz)(z-1)$

Put $z=1$

$2 = 2A$

$\boxed{A=1}$

Put coeff of z^3

$A+B=0$

$1+B=0$

$\boxed{B=-1}$

Coeff of z

$1 = A-C$

$1 = 1-C$

$\boxed{C=0}$

$F(z) = \frac{z}{z-1} - \frac{z^2}{z^2+1}$

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$$Z^{-1}[F(z)] = Z^{-1}\left[\frac{z}{z-1}\right] - Z^{-1}\left[\frac{z^2}{z^2+1}\right]$$

$$= 1^n - \cos\frac{n\pi}{2}$$

$$= 1 - \cos\frac{n\pi}{2}.$$

2. Find the inverse Z-transform of $\frac{z^3}{(z-1)^2(z-2)}$

Let $F(z) = \frac{z^3}{(z-1)^2(z-2)}$

$$\frac{F(z)}{z} = \frac{z^2}{(z-1)^2(z-2)} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z-2} \rightarrow (1)$$

$$\frac{z^2}{(z-1)^2(z-2)} = \frac{A(z-1)(z-2) + B(z-2) + C(z-1)^2}{(z-1)^2(z-2)}$$

$$z^2 = A(z-1)(z-2) + B(z-2) + C(z-1)^2$$

When $z = 1 \Rightarrow 1 = -B \Rightarrow \boxed{B = -1}$

$z = 2 \Rightarrow 4 = C \rightarrow \boxed{C = 4}$

$z = 0 \Rightarrow 2A + (-2B) + C = 0$

$2A + 2 + 4 = 0 \Rightarrow 2A = -6$

$\boxed{A = -3}$

$$(1) \Rightarrow \frac{F(z)}{z} = \frac{-3}{z-1} + \frac{B-1}{(z-1)^2} + \frac{4}{z-2}$$

$$\frac{z^3}{(z-1)^2(z-2)} = -3\frac{z}{z-1} - \frac{z}{(z-1)^2} + 4\frac{z}{z-2}$$

$$Z^{-1}\left[\frac{z^3}{(z-1)^2(z-2)}\right] = -3Z^{-1}\left[\frac{z}{z-1}\right] - Z^{-1}\left[\frac{z}{(z-1)^2}\right] + 4\left[\frac{z}{z-2}\right]$$

$= -3(1) - n + A(2)^n$

$= A \cdot 2^n - 3 - n$

$\frac{z}{z-a} = z(a^n)$

$z(1) = \frac{z}{z-1}$

$\frac{z}{(z-1)^2} = z(n)$

8. Find $z^{-1}\left[\frac{10z}{z^2-3z+2}\right]$

Let $F(z) = \frac{10z}{z^2-3z+2}$

$$\frac{F(z)}{z} = \frac{10}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$\frac{10}{(z-1)(z-2)} = \frac{A(z-2) + B(z-1)}{(z-1)(z-2)}$$

$$10 = A(z-2) + B(z-1)$$

When $z=1$, $\boxed{A=-10}$

$z=2$, $\boxed{B=10}$

$$\therefore \frac{F(z)}{z} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$F(z) = \frac{-10z}{z-1} + \frac{10z}{z-2}$$

$$z^{-1}[F(z)] = -10\, z^{-1}\left[\frac{z}{z-1}\right] + 10\, z^{-1}\left[\frac{z}{z-2}\right]$$

$$= -10(1) + 10(2)^n$$

2. Find $z^{-1}\left[\frac{z^2-3z}{(z-5)(z+2)}\right]$

Let $\frac{F(z)}{z} = \frac{z-3}{(z-5)(z+2)} = \frac{A}{z-5} + \frac{B}{z+2}$ $\rightarrow$ (1)

$$\frac{z-3}{(z-5)(z+2)} = \frac{A(z+2) + B(z-5)}{(z-5)(z+2)}$$

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When $z=-2 \Rightarrow -2-3 = B(-2-5)$

$-5 = -7B$

$\boxed{B = 5/7}$

$z = 5 \Rightarrow 5-3 = A(5+2)$

$2 = 7A$

$\boxed{A = 2/7}$

$① \Rightarrow \frac{F(z)}{z} = \frac{2/7}{z-5} + \frac{5/7}{z+2}$

$F(z) = \frac{2}{7}\cdot\frac{z}{z-5} + \frac{5}{7}\cdot\frac{z}{z+2}$

$z^{-1}[F(z)] = \frac{2}{7} z^{-1}\left[\frac{z}{z-5}\right] + \frac{5}{7} z^{-1}\left[\frac{z}{z+2}\right]$

$= \frac{2}{7}(5)^n + \frac{5}{7}(-2)^n.$