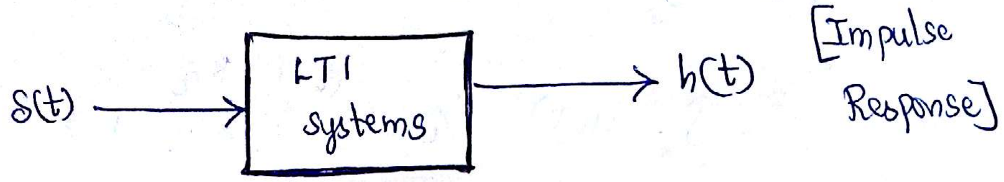
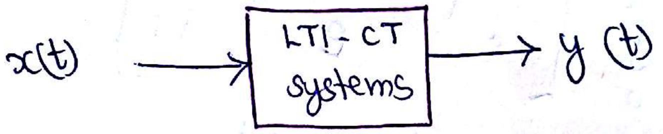
LTI - CONTINUOUS TIME SYSTEMS

Impulse Response :-



Impulse response is the o/p generated by the system when unit impulse is applied at the Input.

Convolution Integral :- Output of an LTI CT system :-



$$y(t) = T[x(t)]$$

$x(t)$ can be represented interms of impulses as

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

$$y(t) = T\left[\int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau \right]$$

$$= \int_{-\infty}^{\infty} x(\tau)\, T[\delta(t-\tau)]\, d\tau$$

$$\therefore\ y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = x(t) * h(t)$$

Relation between I/p and o/p of the system

1) By using convolution Integral obtain the response of the system two unit step input signals :-

$$h(t) = \frac{R}{L} e^{-tR/L} u(t) \quad , \quad x(t) = u(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau) \frac{R}{L} e^{-(t-\tau)R/L} u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} \frac{R}{L} e^{-(t-\tau)R/L} d\tau$$

$$= \int_{0}^{t} \frac{R}{L} e^{-(t-\tau)R/L} d\tau$$

$$= \frac{R}{L} \int_{0}^{t} \left[e^{-tR/L} e^{\tau R/L} d\tau \right]$$

$$= \frac{R}{L} e^{-tR/L} \int_{0}^{t} e^{\tau R/L} d\tau$$

$$= \frac{R}{L} e^{-tR/L} \left[\frac{e^{\tau R/L}}{R/L} \right]_0^t$$

$$= \cancel{\frac{R}{L}} e^{-tR/L} \left[\frac{e^{tR/L}}{\cancel{R/L}} - \frac{1}{R/L} \right]$$

$$y(t) = 1 - e^{-tR/L}$$

Properties of convolution Integral :-

1) commutative property :-

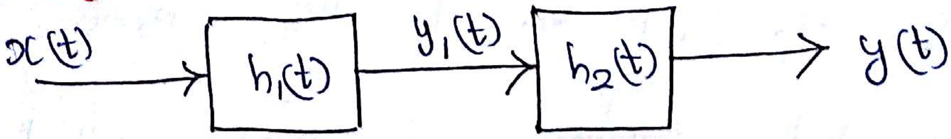
$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

2) Associative Property :-

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$$

L.H.S



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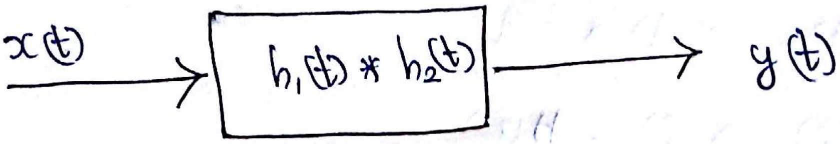
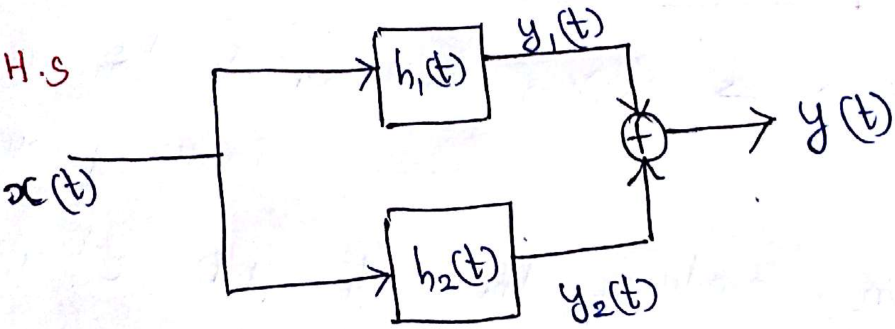


3) Distributive property :-

$$x(t) * h_1(t) + x(t) * h_2(t) = x(t) * [h_1(t) + h_2(t)]$$

L.H.S



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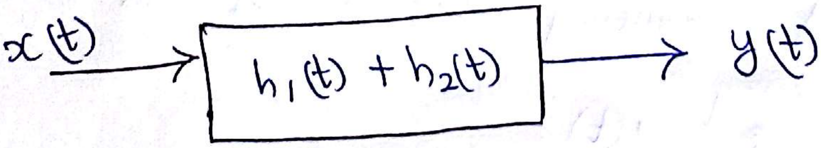

condition for an LTI system to be causal :-

$$h(t) = 0, \quad \text{for } t < 0$$

condition for an LTI system to be stable :-

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$