SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT202 – SIGNALS AND SYSTEMS

II YEAR/ III SEMESTER

UNIT 3 – LTI CONTINUOUS TIME SYSTEMS

TOPIC – LTI SYSTEMS USING LAPLACE TRANSFORM

LAPLACE TRANSFORM

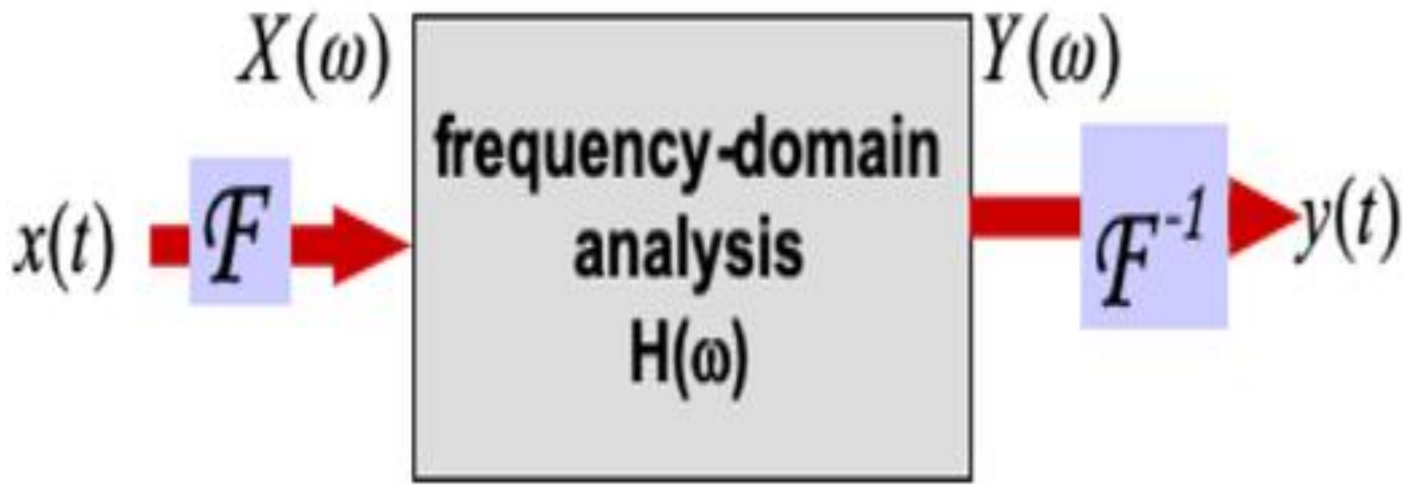
- Laplace Transform represents continuous time signals in terms of complex exponential i.e., e^{-st}
- Laplace transform can be used to analyse signals or functions which are not absolutely integrable
- Continuous time signals can also be analysed effectively using Laplace transforms
- Laplace transform of impulse response is called System Function (or) Transfer Function
- Laplace transform is divided into Unilateral & Bilateral Laplace Transform

- **System Transfer Function:** Ratio of the output to the input.

$$H(s) = \frac{Y(s)}{X(s)}$$

- **Frequency Response:**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



LTI SYSTEM

- Condition for an Linear Time Invariant (LTI) system to be causal:

$$h(t) = 0, \quad t<0$$

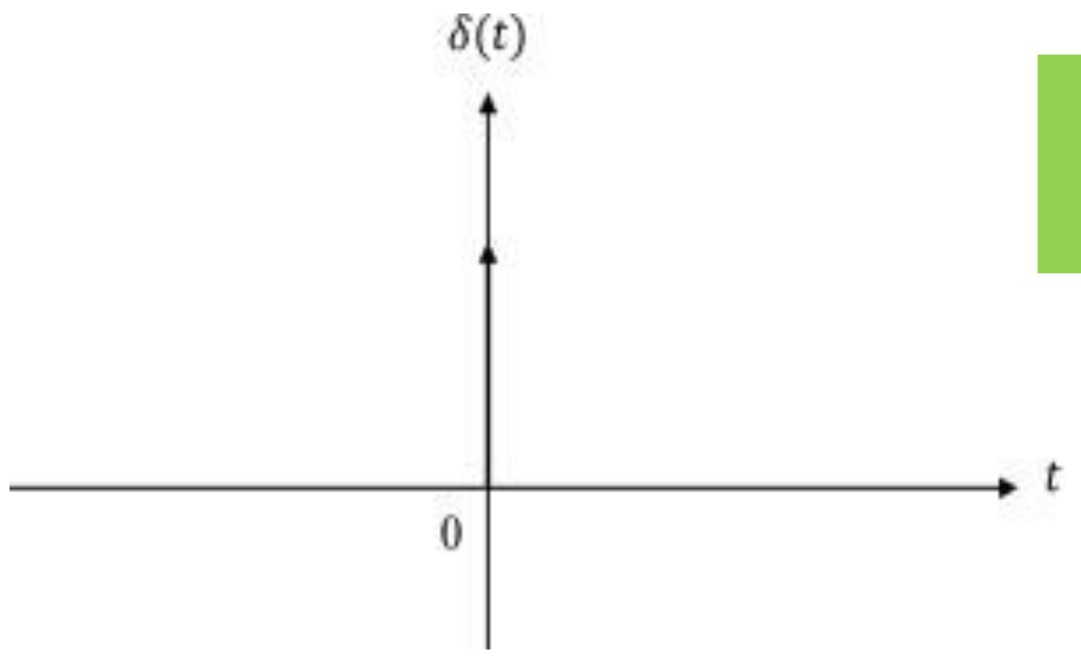
- Condition for an Linear Time Invariant (LTI) system to be stable:

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

IMPULSE RESPONSE

- Impulse response is the output generated by the system, when an unit impulse is applied at the input.

$x(t) = \delta(t)$ → **LTI System** → $y(t) = h(t)$

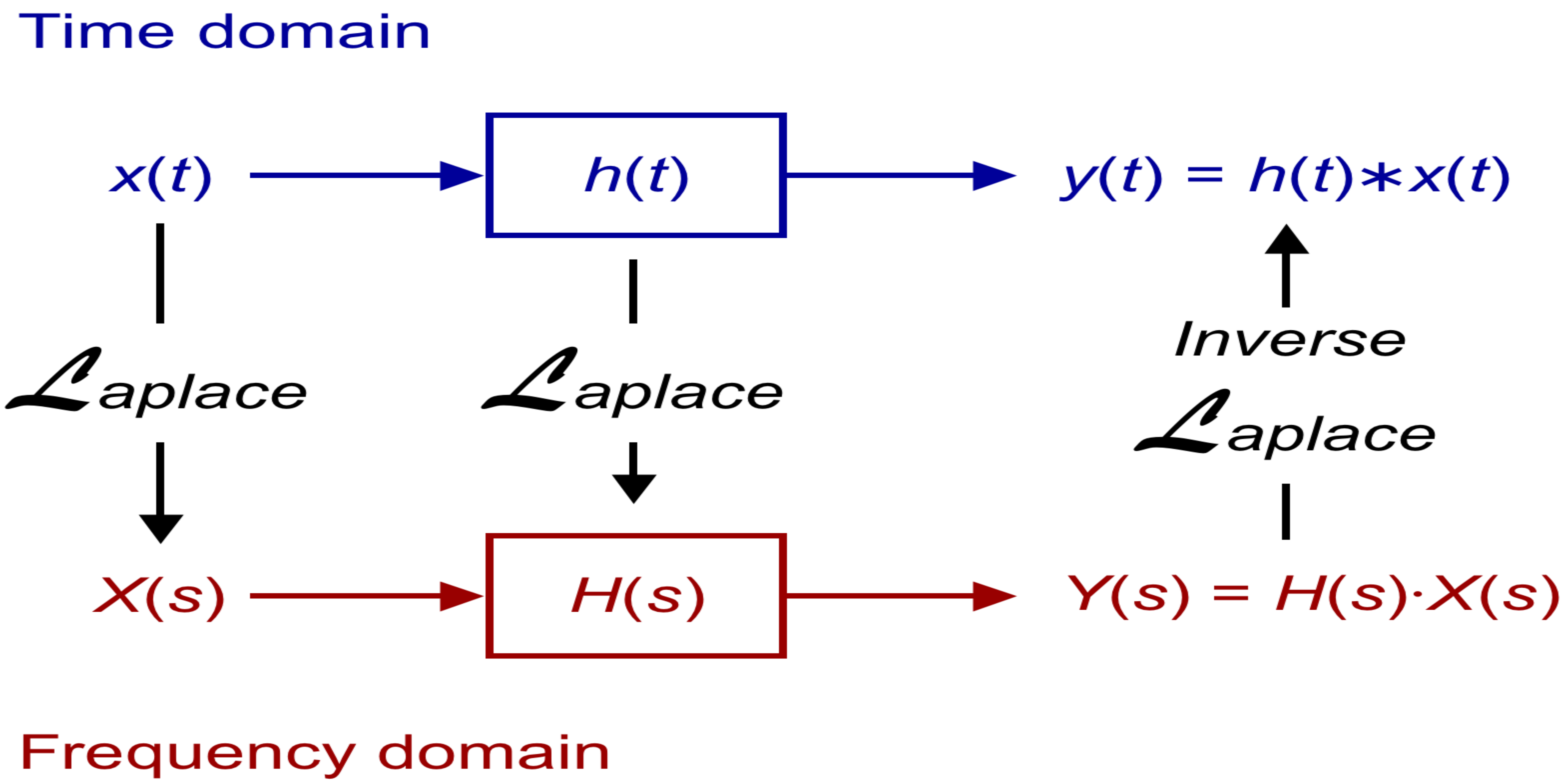


$\delta(t) = 1$ for $t = 0$
$= 0$ for $t \neq 0$

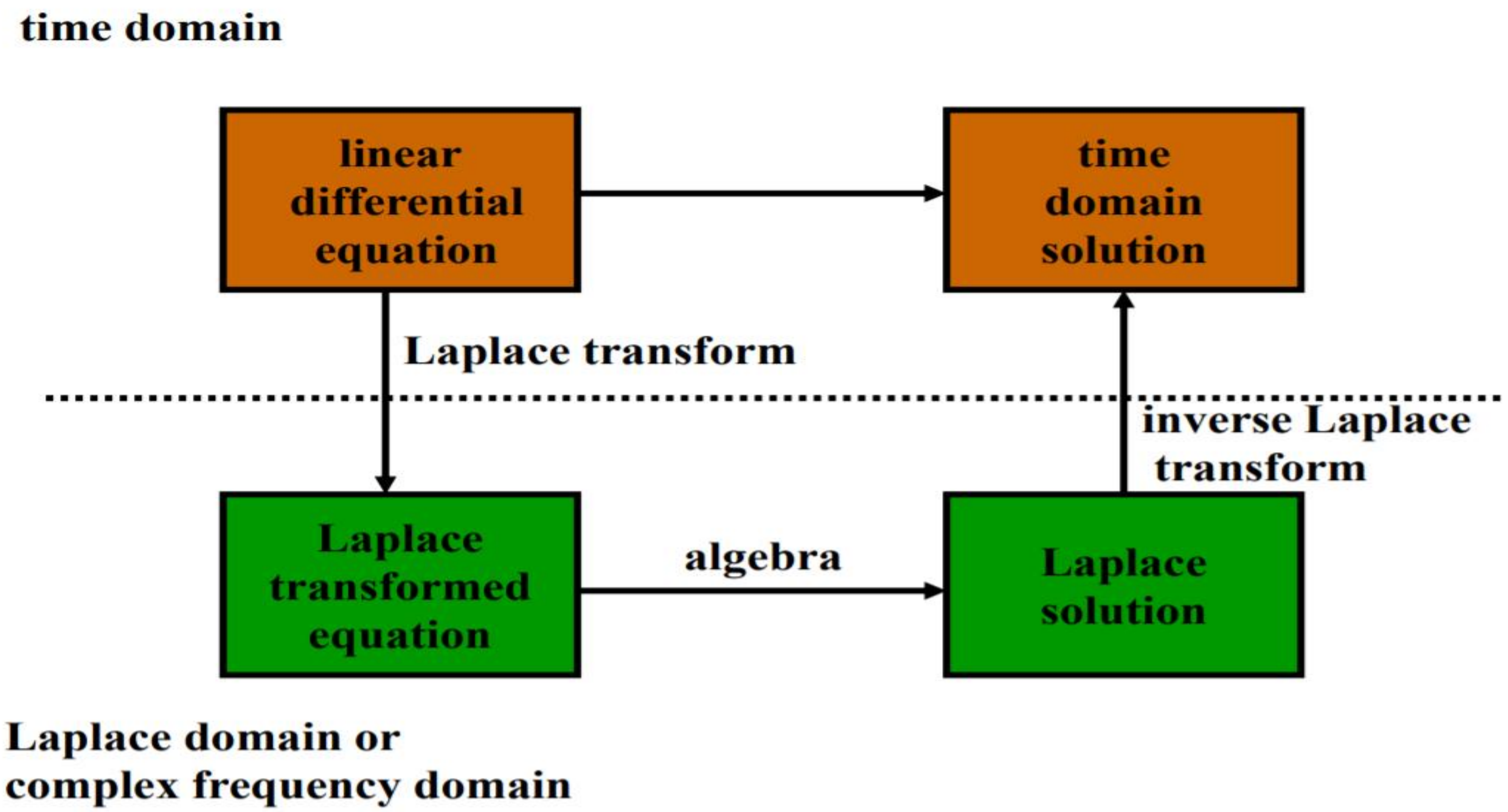
- $H(s) = \frac{Y(s)}{X(s)}$

- $h(t) = L^{-1}\{H(s)\}$

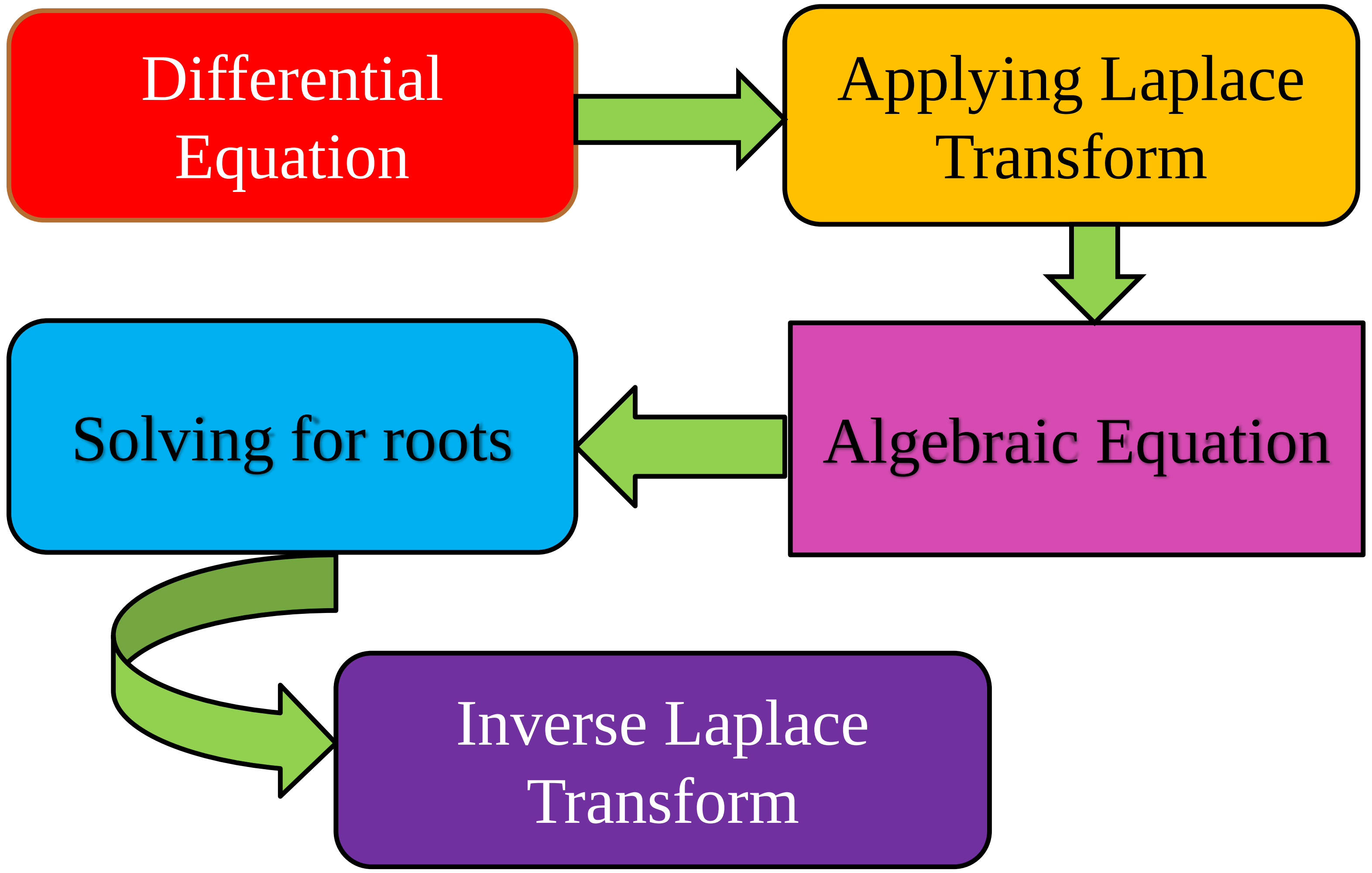
TIME DOMAIN INTO FREQUENCY DOMAIN



TIME DOMAIN INTO FREQUENCY DOMAIN

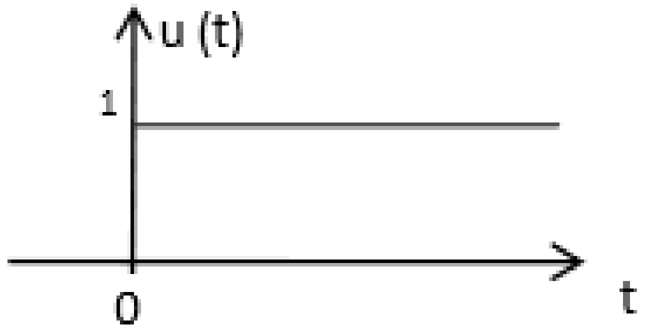


TO FIND IMPULSE RESPONSE

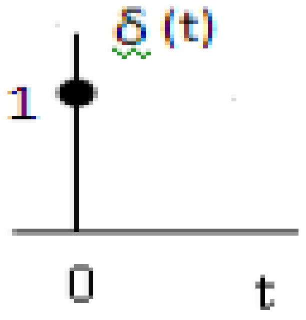


BASIC SIGNALS

$$u(t) = 1 \text{ for } t \geq 0$$
$$= 0 \text{ for } t < 0$$



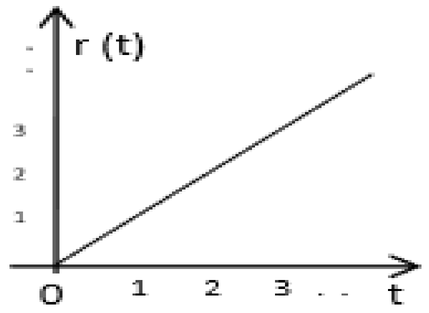
Unit step signal



Unit Impulse signal

$$\delta(t) = 1 \text{ for } t = 0$$
$$= 0 \text{ for } t \neq 0$$

$$r(t) = t \text{ for } t \geq 0$$
$$= 0 \text{ for } t < 0$$



Unit ramp signal

LAPLACE TRANSFORM RESULTS

Laplace Transform of $x(t) = u(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} 1\, e^{-st}\, dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$X(s) = \frac{1}{s}$$

$x(t) = r(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} t\, e^{-st}\, dt$$

$$= \left[\frac{t\, e^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^{\infty}$$

$$X(s) = \frac{1}{s^2}$$

LAPLACE TRANSFORM RESULTS

Find the o/p of the system :- $h(t) = u(t)$, $x(t) = e^{-2t}u(t)$

$H(S) = \frac{1}{S}$ $\quad X(S) = \frac{1}{S+2}$

$H(S) = \frac{Y(S)}{X(S)}$

$Y(S) = H(S) \cdot X(S)$

$= \frac{1}{S} \cdot \frac{1}{S+2}$

$= \frac{A}{S} + \frac{B}{S+2} \Rightarrow A(S+2) + B(S)$

$S = 0$

$1 = A(2)$

$\boxed{A = 1/2}$

$S = -2$

$1 = A(0) + B(-2)$

$\boxed{B = -1/2}$

$Y(S) = \frac{1}{2(S)} - \frac{1}{2(S+2)}$

$= \frac{1}{2} L^{-1}\left(\frac{1}{S}\right) - \frac{1}{2} L^{-1}\left(\frac{1}{S+2}\right)$

$y(t) = \frac{1}{2} u(t) - \frac{1}{2} e^{-2t} u(t)$

LAPLACE TRANSFORM RESULTS

$$RC\frac{d}{dt}y(t) + y(t) = x(t)$$ → Find Impulse Response

Apply Inverse Laplace Transform

$$RC\,S\,Y(S) + Y(S) = X(S)$$

$$\therefore H(S) = \frac{Y(S)}{X(S)} = \frac{1}{RCS+1}$$

Impulse Response :-

$$H(S) = \frac{1}{RCS+1}$$

$$\therefore h(t) = \frac{1}{RC}e^{-t}u(t)$$

TO FIND IMPULSE RESPONSE

$$\frac{d^2}{dt^2} y(t) - \frac{d}{dt} y(t) - 2y(t) = x(t)$$

$$s^2 Y(s) - sY(s) - 2Y(s) = X(s)$$

$$(s^2 - s - 2) Y(s) = X(s)$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$$

$$\frac{A}{s-2} + \frac{B}{s+1} = \frac{1}{(s-2)(s+1)}$$

$$1 = A(s+1) + B(s-2)$$

$s = -1$

$B(-3) = 1$

$$\boxed{B = -1/3}$$

put $s = 2$

$3A = 1$

$$\boxed{A = 1/3}$$

$$H(s) = \frac{1}{3(s-2)} - \frac{1}{3(s+1)}$$

$$= \frac{1}{3} L^{-1}\left(\frac{1}{s-2}\right) - \frac{1}{3} L^{-1}\left(\frac{1}{s+1}\right)$$

$$h(t) = \frac{1}{3} e^{2t} u(t) - \frac{1}{3} e^{-t} u(t)$$

LAPLACE TRANSFORM RESULTS

$$e^{-t}u(t) \xleftrightarrow{LT} \frac{1}{s+1}, \quad \mathrm{Re}\{s\} > -1.$$

$$-e^{-2t}u(t) \xleftrightarrow{LT} -\frac{1}{s+2}, \quad \mathrm{Re}\{s\} > -2$$

$$e^{-t}u(t) - e^{-2t}u(t) \xleftrightarrow{LT} \frac{1}{s+1} - \frac{1}{s+2}, \quad \mathrm{Re}\{s\} > -1$$

$$e^{-t}u(t) - e^{-2t}u(t) \xleftrightarrow{LT} \frac{1}{(s+1)(s+2)}, \quad \mathrm{Re}\{s\} > -1$$

Shifting Property – Unilateral Laplace Transform

$$L[(d/dt)\, x(t)] = S\,X(S) - x(0^-)$$

$$L[(d^2/dt^2)\, x(t)] = S^2\,X(S) - S\,x(0^-) - x'(0^-)$$

$$L[(d^3/dt^3)\, x(t)] = S^3\,X(S) - S^2\,x(0^-) - S\,x'(0^-) - x''(0^-)$$

TO FIND IMPULSE RESPONSE

Solve using Differential Equation $\frac{d}{dt}y(t) + 5y(t) = x(t)$ with initial condition $y(0^-) = -2$ and i/p $x(t) = 3e^{-2t}u(t)$

$$\frac{d}{dt}y(t) + 5y(t) = x(t)$$

$$sY(s) - y(0^-) + 5Y(s) = X(s)$$

$$sY(s) + 2 + 5Y(s) = \frac{3}{s+2}$$

$$Y(s)[s+5] + 2 = \frac{3}{s+2}$$

$$Y(s)[s+5] = \frac{3}{s+2} - 2$$

$$Y(s) = \frac{3}{(s+2)[s+5]} - \frac{2}{(s+5)}$$

$$\frac{3}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

$$3 = A(s+5) + B(s+2)$$

put $s = -5$: $3 = B(-3)$, $B = -1$

put $s = -2$: $3 = A(3)$, $A = 1$

$$Y(s) = \left[\frac{1}{s+2} - \frac{1}{s+5}\right] - \frac{2}{s+5}$$

$$Y(s) = \frac{1}{s+2} - \frac{3}{s+5}$$

$$\therefore y(t) = e^{-2t}u(t) - 3e^{-5t}u(t)$$

SYSTEM TRANSFER FUNCTION

The input output relation of a system at initial rest is given by $\frac{d^2}{dt^2} y(t) + 4 \frac{d}{dt} y(t) + 3y(t) = \frac{d}{dt} x(t) + 2x(t)$. Find system transfer function

$$s^2 Y(s) + 4sY(s) + 3Y(s) = sX(s) + 2X(s)$$

$$Y(s)\left[s^2+4s+3\right] = X(s)\left[s+2\right]$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+2}{s^2+4s+3}$$

Freq Response :-

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega+2}{(j\omega)^2+4j\omega+3}$$

TO FIND IMPULSE RESPONSE

The input output relation of a system at initial rest is given by $\frac{d^2}{dt^2} y(t) + 4 \frac{d}{dt} y(t) + 3y(t) = \frac{d}{dt} x(t) + 2x(t)$

$$H(s) = \frac{s+2}{s^2+4s+3}$$

$$\frac{s+2}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

$$s+2 = A(s+1) + B(s+3)$$

sub $s=-1$

$1 = 2B$

$\boxed{B = 1/2}$

Sub $s=-3$

$-1 = -2A$

$\boxed{A = 1/2}$

$$H(s) = \frac{1}{2(s+3)} + \frac{1}{2(s+1)}$$

$$h(t) = \frac{1}{2} L^{-1}\left(\frac{1}{s+3}\right) + \frac{1}{2} L^{-1}\left(\frac{1}{s+1}\right)$$

$$h(t) = 1/2\, e^{-3t} u(t) + \frac{1}{2} e^{-t} u(t)$$

ASSESSMENT

1. What is meant by impulse response?
2. Define Unit step and Unit Impulse Signal.
3. The condition of an LTI system to be causal is given by --------------
4. The system transfer function is given by --------------
5. Laplace transform of Unit step function is given by -----------------
6. Laplace Transform of unit ramp function is given by ---------------

THANK YOU