Formation of Difference Equations

(1) Form the difference equation from $y_n = A5^n$

Now $y_n = A5^n$

$y_{n+1} = A5^{n+1}$

$= A5^n \cdot 5$

$= 5A5^n$

$= 5y_n$

$\Rightarrow y_{n+1} - 5y_n = 0$ is the required difference equations

(2) Form the difference equation from the relation $y_n = A2^n + B3^n$

Now $y_n = A2^n + B3^n$ ——(1)

$y_{n+1} = A2^{n+1} + B3^{n+1} \Rightarrow y_{n+1} = 2A2^n + 3B3^n$ ——(2)

$y_{n+2} = A2^{n+2} + B3^{n+2} \Rightarrow y_{n+2} = 4A2^n + 9B3^n$ ——(3)

(1) $\times 2 \Rightarrow 2y_n = 2A2^n + 2B3^n$ ——(4)

(2) $\Rightarrow y_{n+1} = 2A2^n + 3B3^n$ ——(2) (−) (−)

$2y_n - y_{n+1} = -B3n$ ——(a)

(2) × 2 ⇒ $2y_{n+1} = 4A2^n + 6B3^n$ ——— (5)

(3) ⇒ $y_{n+2} = 4A2^n + 9B3^n$ ——— (3)

$2y_{n+1} - y_{n+2} = -3B3^n$ ——— (6)

Equate (4) & (6)

$3[2y_n - y_{n+1}] = 3[2y_{n+1} - y_{n+2}]$

$y_{n+2} - 5y_{n+1} + 6y_n = 0$ is the required difference equation.

(3) Find the difference equation from $y(n) = (A + nB)2^n$

$y(n) = (A + nB)2^n$

(a) $y(n) = A2^n + nB2^n$ ——— (1)

$y(n+1) = A2^{n+1} + (n+1)B2^{n+1} \Rightarrow y(n+1) = 2A2^n + (n+1)2B2^n$ ——— (2)

$y(n+2) = A2^{n+2} + (n+2)B2^{n+2} \Rightarrow y(n+2) = 4A2^n + (n+2)4B2^n$ ——— (3)

$(1) \times 2 \Rightarrow 2y(n) = 2A2^n + 2nB2^n$ ——— (4)

$(2) \Rightarrow y(n+1) = 2A2^n + 2(n+1)B2^n$

$2y(n) - y(n+1) = 2B2^n [n - n - 1]$

$2y(n) - y(n+1) = -2B2^n$ ——— (a)

$(3) \times 2 \Rightarrow 2y(n+1) = 4A2^n + 4(n+1)B2^n$ ——— (5)

$(5) \Rightarrow y(n+2) = 4A2^n + 4(n+2)B2^n$

$2y(n+1) - y(n+2) = 4B2^n [n+1 - n - 2]$

$2y(n+1) - y(n+2) = -4B2^n$ ——— (b)

Equate (a) & (b)

$2y(n+1) - y(n+2) = 2[2y(n) - y(n+1)]$

$\Rightarrow y(n+2) - 4y(n+1) + 4y(n) = 0$ is the required difference equation.