SNS COLLEGE OF TECHNOLOGY

AN AUTONOMOUS INSTITUTION
ACCREDITED BY NBA – AICTE AND ACCREDITED BY NAAC – UGC WITH 'A' GRADE APPROVED BY AICTE, NEW DELHI & AFFILIATED TO ANNA UNIVERSITY, CHENNAI

COURSE NAME : 23GET275 VQAR-I
II YEAR /III SEMESTER

Logical Series,Coding & Decoding

What is a SEQUENCE?

- In mathematics, a **sequence** is an ordered list. Like a set, it contains members (also called *elements*, or *terms*). The number of ordered elements (possibly infinite) is called the *length* of the sequence. Unlike a set, order matters, and exactly the same elements can appear multiple times at different positions in the sequence. Most precisely, a sequence can be defined as a function whose domain is a countable totally ordered set, such as the natural numbers.

Source: Wikipedia:http://en.wikipedia.org/wiki/Sequence

INFINITE SEQUENCE

- An infinite sequence is a function with domain the set of natural numbers $N = \{1, 2, 3, \dots \dots \dots\}$.
- For example, consider the function "a" defined by

$$a(n) = n^2 \quad (n = 1, 2, 3, \dots \dots)$$

Instead of the usual functional notation $a(n)$, for sequences we usually write

$$a_n = n^2$$

That is, a letter with a subscript, such as a_n, is used to represent numbers in the range of a sequence. For the sequence defined by $a_n = n^2$,

$$a_1 = 1^2 = 1$$

$$a_2 = 2^2 = 4$$

$$a_3 = 3^2 = 9$$

$$a_4 = 4^2 = 16$$

General or nth Term

- A sequence is frequently defined by giving its range. The sequence on the given example can be written as

$$1, 4, 9, 16, \dots \dots \dots, n^2, \dots \dots$$

Each number in the range of a sequence is a ***term*** of the sequence, with a_n the ***nth term*** or ***general term*** of the sequence. The formula for the nth term generates the terms of a sequence by repeated substitution of counting numbers for n.

FINITE SEQUENCE

- A finite sequence with m terms is a function with domain the set of natural numbers $\{1, 2,3, \dots \dots, m\}$
- For example, $2, 4, 6, 8, 10$ is a finite sequence with 5 terms where $a_n = 2n$, for $n = 1,2,3,4,5$. In contrast, $2,4,6,8,10, \dots \dots$ is an infinite sequence where $a_n = 2n$, for $n = 1,2,3,4,5 \dots \dots.$

Sample Problems

1. Write the first five terms of the infinite sequence with general term $a_n = 2n - 1$.

Answer:

$$a_1 = 2(1) - 1 = 1$$
$$a_2 = 2(2) - 1 = 3$$
$$a_3 = 2(3) - 1 = 5$$
$$a_4 = 2(4) - 1 = 7$$
$$a_5 = 2(5) - 1 = 9$$

Thus, the first five terms are 1,3,5,7,9 and the sequence is

$$1, 3, 5, 7, 9, \ldots \ldots 2n - 1, \ldots \ldots$$

2. A finite sequence has four terms, and the formula for the nth term is $x_n = (-1)^n \frac{1}{2^{n-1}}$. What is the sequence?

Answer:

$$x_1 = (-1)^1 \frac{1}{2^{1-1}} = -1$$

$$x_2 = (-1)^2 \frac{1}{2^{2-1}} = \frac{1}{2}$$

$$x_3 = (-1)^3 \frac{1}{2^{3-1}} = -\frac{1}{4}$$

$$x_4 = (-1)^4 \frac{1}{2^{4-1}} = \frac{1}{8}$$

Thus the sequence is

3. Find a formula for a_n given the first few terms of the sequence.

a.) 2, 3, 4, 5, …

Answer: Each term is one larger than the corresponding natural number. Thus, we have,

$$a_1 = 2 = 1 + 1$$
$$a_2 = 3 = 2 + 1$$
$$a_3 = 4 = 3 + 1$$
$$a_4 = 5 = 4 + 1$$

Hence, $a_n = n + 1$

b.) 3, 6, 9, 12, ……

Answer:

$$a_1 = 3 = 3 \cdot 1$$

$$a_2 = 6 = 3 \cdot 2$$

$$a_3 = 9 = 3 \cdot 3$$

$$a_4 = 12 = 3 \cdot 4$$

Thus, $a_n = 3n$

SERIES

- Associated with every sequence, is a ***SERIES***, the indicated sum of the sequence.
- For example, associated with the sequence 2, 4, 6, 8, 10, is the series 2+4+6+8+10 and associated with the sequence -1, ½, -1/4, 1/8, is the series (-1)+(1/2)+(-1/4)+(1/8).

Sigma Summation Notation

- The Greek letter $\sum$ (sigma) is often used as a summation symbol to abbreviate a series.
- The series 2 + 4 +6 + 8 + 10 which has a general term $x_n = 2n$, can be written as

$$\sum_{n=1}^{5} x_n \quad or \sum_{n=1}^{5} 2n$$

and is read as "the sum of the terms x_n or $2n$ as n varies from 1 to 5". The letter n is the index on the summation while 1 and 5 are the lower and upper limits of summation, respectively.

- In general, if $x_1, x_2, x_3, x_4 \ldots, x_n$ is a sequence associated with a series of

$$\sum_{k=1}^{n} x_k = x_1 + x_2 + x_3 + x_4 + \cdots + x_n$$

Sample Problem

1. Write out the series

$$\sum_{k=1}^{5}(k^2+1)$$

without using the sigma summation notation.

Answer:

$$x_1 = (1)^2 + 1 = 1 + 1 = 2$$
$$x_2 = (2)^2 + 1 = 4 + 1 = 5$$
$$x_3 = (3)^2 + 1 = 9 + 1 = 10$$
$$x_4 = (4)^2 + 1 = 16 + 1 = 17$$
$$x_5 = (5)^2 + 1 = 25 + 1 = 26$$

Thus,

$$\sum_{k=1}^{5}(k^2+1) = 2 + 5 + 10 + 17 + 26 = 60$$

2. Express and write the series

$$\sum_{k=2}^{4} (-1)^k \sqrt{k+1}$$

without using sigma notation.

Answer:

$$x_2 = (-1)^2\sqrt{2+1} = \sqrt{3}$$
$$x_3 = (-1)^3\sqrt{3+1} = -2$$
$$x_4 = (-1)^4\sqrt{4+1} = \sqrt{5}$$

Thus,

$$\sum_{k=2}^{4} (-1)^k \sqrt{k+1} = \sqrt{3} - 2 + \sqrt{5}$$

Break a leg!

1. Find the first four terms and the seventh term ($n = 1,2,3,4$ &7) if the general term of the sequence is $x_n = \frac{(-1)^n}{n}$.
2. Find a formula for a_n given the few terms of the sequence. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

PROF. DENMAR ESTRADA MARASIGAN

THANK YOU VERY MUCH!!!

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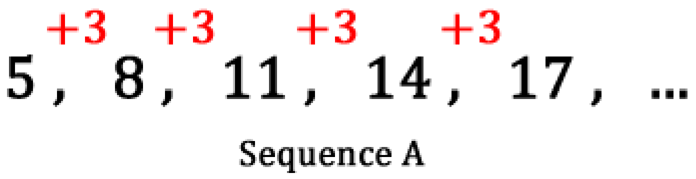
Sequences and Seri

- A sequence is a list of numbers in an speci
- A series is the sum of the numbers in a sequence.
- In this presentation, we will see two types of sequences and
nd geometric.



Sequences

- A sequence of numbers is a list of numbers arranged in a specific order. Consider the list of numbers given below, which is a sequence.

- 5, 10, 15, 20, 25, 30, …

- Each number is called a **term** of the sequence. The first term of the sequence is 5. we indicate this by writing $a_1=5$. Since the second term is 10, $a_{2.}$

- The three dots, called an **ellipsis**, indicate that the sequence continues indefintely and is an **infinite sequence.**

- An **infinite sequence** is a function whose domain is the set of natural numbers.

- Note that the terms of the sequence 5, 10, 15, 20, 25, 30, … are found by multiplying each natural number by 5. The **general term of the sequence**, a_n which defines the sequence is $a_n = 5n$.

- A **finite sequence** is a function whose domain includes only the first *n* natural numbers. A finite sequence has only a finite numbers of terms.

- Example of Finite Sequence:
- 5, 10, 15, 20
- 2, 4, 8, 16, 20

- There exists another distinction between sequences, *increasing*, *decreasing* and *alternate.*
- **Increasing** sequences are the ones that each term is greater tan the preceding term, while the **decreasing** sequences each term is less than the preceding term.

- In an **alternate** sequence, the terms alternate signs.

Series

- A **series** is the expressed sum of the terms of a sequence.

- It may be finite or infinite, depending on whether the sequence it is based on, is finite or infinite.

- Examples
- **Finite sequences**
- A_1, A_2, A_3

Infinite sequences

$A_1, A_2, A_3 \ldots$

- **Finite series**
- $A_1 + A_2 + A_3$

Infinite series

$A_1 + A_2 + A_3 \ldots$

Find Partial Sums and Summation Notation Σ

- For an infinite sequence with the terms A_1, A_2, A_3..., a **partial sum** is the sum of a finite number of consecutive terms of the sequence, beginnig with the first term.

- $s_1 = a_1$
- $s_2 = a_1 + a_2$
- $s_3 = a_1 + a_2 + a_3$
- $\vdots$
- $S_n = a_1 + a_2 + a_3 + \dots + a_n$

- The sum of the first *n* terms of the sequence whose *n*th term is a_n is represented by

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots + x_n$$

- Where *i* is called the **index of summation** or simply the **index**, *n* is the **upper limit of summation**, and 1 is the **lower limit of summation**.