Resolution

Resolution is a theorem proving technique that proceeds by building refutation proofs, i.e., proofs by contradictions. It was invented by a Mathematician John Alan Robinson in the year 1965.

Resolution is used, if there are various statements are given, and we need to prove a conclusion of those statements. Unification is a key concept in proofs by resolutions. Resolution is a single inference rule which can efficiently operate on the **conjunctive normal form or clausal form**.

Clause: Disjunction of literals (an atomic sentence) is called a **clause**. It is also known as a unit clause.

Conjunctive Normal Form: A sentence represented as a conjunction of clauses is said to be **conjunctive normal form** or **CNF**.

Steps for Resolution:

1. Conversion of facts into first-order logic.
2. Convert FOL statements into CNF
3. Negate the statement which needs to prove (proof by contradiction)
4. Draw resolution graph (unification).

To better understand all the above steps, we will take an example in which we will apply resolution.

Example:

1. **John likes all kind of food.**
2. **Apple and vegetable are food**
3. **Anything anyone eats and not killed is food.**
4. **Anil eats peanuts and still alive**
5. **Harry eats everything that Anil eats.**
   Prove by resolution that:
6. **John likes peanuts.**

Step-1: Conversion of Facts into FOL

In the first step we will convert all the given statements into its first order logic.

a. ∀x: food(x) → likes(John, x)

b. food(Apple) Λ food(vegetables)

c. ∀x ∀y: eats(x, y) Λ ¬ killed(x) → food(y)

d. eats (Anil, Peanuts) Λ alive(Anil).

e. ∀x : eats(Anil, x) → eats(Harry, x)

f. ∀x: ¬ killed(x) → alive(x) } **added predicates.**

g. ∀x: alive(x) → ¬ killed(x)

h. likes(John, Peanuts)

Step-2: Conversion of FOL into CNF

In First order logic resolution, it is required to convert the FOL into CNF as CNF form makes easier for resolution proofs.

- **Eliminate all implication (→) and rewrite**
  - ∀x ¬ food(x) V likes(John, x)
  - food(Apple) Λ food(vegetables)
  - ∀x ∀y ¬ [eats(x, y) Λ ¬ killed(x)] V food(y)
  - eats (Anil, Peanuts) Λ alive(Anil)
  - ∀x ¬ eats(Anil, x) V eats(Harry, x)
  - ∀x¬ [¬ killed(x)] V alive(x)
  - ∀x ¬ alive(x) V ¬ killed(x)
  - likes(John, Peanuts).
- **Move negation (¬)inwards and rewrite**
  - ∀x ¬ food(x) V likes(John, x)
  - food(Apple) Λ food(vegetables)
  - ∀x ∀y ¬ eats(x, y) V killed(x) V food(y)
  - eats (Anil, Peanuts) Λ alive(Anil)
  - ∀x ¬ eats(Anil, x) V eats(Harry, x)
  - ∀x ¬killed(x)] V alive(x)
  - ∀x ¬ alive(x) V ¬ killed(x)
  - likes(John, Peanuts).
- **Rename variables or standardize variables**
  - ∀x ¬ food(x) V likes(John, x)
  - food(Apple) Λ food(vegetables)
  - ∀y ∀z ¬ eats(y, z) V killed(y) V food(z)
  - eats (Anil, Peanuts) Λ alive(Anil)
  - ∀w¬ eats(Anil, w) V eats(Harry, w)
  - ∀g ¬killed(g)] V alive(g)
  - ∀k ¬ alive(k) V ¬ killed(k)
  - likes(John, Peanuts).
- **Eliminate existential instantiation quantifier by elimination.** In this step, we will eliminate existential quantifier ∃, and this process is known as **Skolemization**. But in this example problem since there is no existential quantifier so all the statements will remain same in this step.
- **Drop Universal quantifiers.** In this step we will drop all universal quantifier since all the statements are not implicitly quantified so we don't need it.
  - ¬ food(x) V likes(John, x)
  - food(Apple)
  - food(vegetables)
  - ¬ eats(y, z) V killed(y) V food(z)
  - eats (Anil, Peanuts)
  - alive(Anil)
  - ¬ eats(Anil, w) V eats(Harry, w)
  - killed(g) V alive(g)

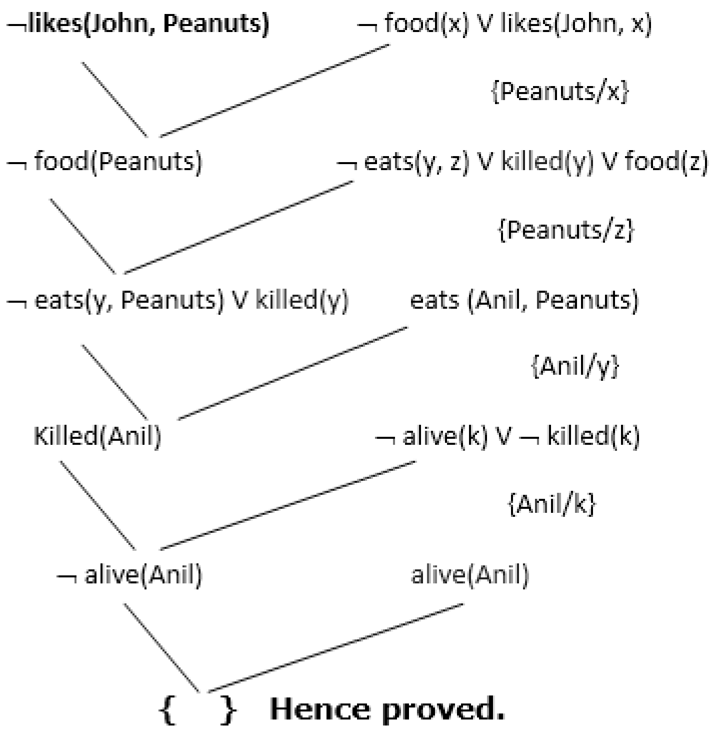
    - ¬ alive(k) V ¬ killed(k)
    - likes(John, Peanuts).
- **Distribute conjunction ∧ over disjunction ¬.** This step will not make any change in this problem.

Step-3: Negate the statement to be proved

In this statement, we will apply negation to the conclusion statements, which will be written as ¬likes(John, Peanuts)

Step-4: Draw Resolution graph:

Now in this step, we will solve the problem by resolution tree using substitution. For the above problem, it will be given as follows:



Explanation of Resolution graph:

- In the first step of resolution graph, **¬likes(John, Peanuts)** , and **likes(John, x)** get resolved(canceled) by substitution of **{Peanuts/x}**, and we are left with **¬ food(Peanuts)**
- In the second step of the resolution graph, **¬ food(Peanuts)** , and **food(z)** get resolved (canceled) by substitution of **{ Peanuts/z}**, and we are left with **¬ eats(y, Peanuts) V killed(y)** .
- In the third step of the resolution graph, **¬ eats(y, Peanuts)** and **eats (Anil, Peanuts)** get resolved by substitution **{Anil/y}**, and we are left with **Killed(Anil)** .
- In the fourth step of the resolution graph, **Killed(Anil)** and **¬ killed(k)** get resolve by substitution **{Anil/k}**, and we are left with **¬ alive(Anil)** .
- In the last step of the resolution graph **¬ alive(Anil)** and **alive(Anil)** get resolved.

Resolution Exercise Solutions

2. Consider the following axioms:

1. Every child loves Santa.
   $\forall x\ (CHILD(x) \rightarrow LOVES(x,Santa))$
2. Everyone who loves Santa loves any reindeer.
   $\forall x\ (LOVES(x,Santa) \rightarrow \forall y\ (REINDEER(y) \rightarrow LOVES(x,y)))$
3. Rudolph is a reindeer, and Rudolph has a red nose.
   $REINDEER(Rudolph) \land REDNOSE(Rudolph)$
4. Anything which has a red nose is weird or is a clown.
   $\forall x\ (REDNOSE(x) \rightarrow WEIRD(x) \lor CLOWN(x))$
5. No reindeer is a clown.
   $\neg \exists x\ (REINDEER(x) \land CLOWN(x))$
6. Scrooge does not love anything which is weird.
   $\forall x\ (WEIRD(x) \rightarrow \neg LOVES(Scrooge,x))$
7. (Conclusion) Scrooge is not a child.
   $\neg CHILD(Scrooge)$

3. Consider the following axioms:

1. Anyone who buys carrots by the bushel owns either a rabbit or a grocery store.
   $\forall x\ (BUY(x) \rightarrow \exists y\ (OWNS(x,y) \land (RABBIT(y) \lor GROCERY(y))))$
2. Every dog chases some rabbit.
   $\forall x\ (DOG(x) \rightarrow \exists y\ (RABBIT(y) \land CHASE(x,y)))$
3. Mary buys carrots by the bushel.
   $BUY(Mary)$
4. Anyone who owns a rabbit hates anything that chases any rabbit.
   $\forall x\ \forall y\ (OWNS(x,y) \land RABBIT(y) \rightarrow \forall z\ \forall w\ (RABBIT(w) \land CHASE(z,w) \rightarrow HATES(x,z)))$
5. John owns a dog.
   $\exists x\ (DOG(x) \land OWNS(John,x))$
6. Someone who hates something owned by another person will not date that person.
   $\forall x\ \forall y\ \forall z\ (OWNS(y,z) \land HATES(x,z) \rightarrow \neg DATE(x,y))$
7. (Conclusion) If Mary does not own a grocery store, she will not date John.
   $((\neg \exists x\ (GROCERY(x) \land OWN(Mary,x))) \rightarrow \neg DATE(Mary,John))$

4. Consider the following axioms:

1. Every Austinite who is not conservative loves some armadillo.
   $\forall x\ (AUSTINITE(x) \wedge \neg\ CONSERVATIVE(x) \rightarrow \exists y\ (ARMADILLO(y) \wedge LOVES(x,y)))$
2. Anyone who wears maroon-and-white shirts is an Aggie.
   $\forall x\ (WEARS(x) \rightarrow AGGIE(x))$
3. Every Aggie loves every dog.
   $\forall x\ (AGGIE(x) \rightarrow \forall y\ (DOG(y) \rightarrow LOVES(x,y)))$
4. Nobody who loves every dog loves any armadillo.
   $\neg\ \exists x\ ((\forall y\ (DOG(y) \rightarrow LOVES(x,y))) \wedge \exists z\ (ARMADILLO(z) \wedge LOVES(x,z)))$
5. Clem is an Austinite, and Clem wears maroon-and-white shirts.
   $AUSTINITE(Clem) \wedge WEARS(Clem)$
6. (Conclusion) Is there a conservative Austinite?
   $\exists x\ (AUSTINITE(x) \wedge CONSERVATIVE(x))$

5. Consider the following axioms:

1. Anyone whom Mary loves is a football star.
   $\forall x\ (LOVES(Mary,x) \rightarrow STAR(x))$
2. Any student who does not pass does not play.
   $\forall x\ (STUDENT(x) \wedge \neg\ PASS(x) \rightarrow \neg\ PLAY(x))$
3. John is a student.
   $STUDENT(John)$
4. Any student who does not study does not pass.
   $\forall x\ (STUDENT(x) \wedge \neg\ STUDY(x) \rightarrow \neg\ PASS(x))$
5. Anyone who does not play is not a football star.
   $\forall x\ (\neg\ PLAY(x) \rightarrow \neg\ STAR(x))$
6. (Conclusion) If John does not study, then Mary does not love John.
   $\neg\ STUDY(John) \rightarrow \neg\ LOVES(Mary,John)$

6. Consider the following axioms:

1. Every coyote chases some roadrunner.
   $\forall x\ (COYOTE(x) \rightarrow \exists y\ (RR(y) \wedge CHASE(x,y)))$
2. Every roadrunner who says ``beep-beep'' is smart.
   $\forall x\ (RR(x) \wedge BEEP(x) \rightarrow SMART(x))$
3. No coyote catches any smart roadrunner.
   $\neg\ \exists x\ \exists y\ (COYOTE(x) \wedge RR(y) \wedge SMART(y) \wedge CATCH(x,y))$
4. Any coyote who chases some roadrunner but does not catch it is frustrated.
   $\forall x\ (COYOTE(x) \wedge \exists y\ (RR(y) \wedge CHASE(x,y) \wedge \neg\ CATCH(x,y)) \rightarrow FRUSTRATED(x))$
5. (Conclusion) If all roadrunners say ``beep-beep'', then all coyotes are frustrated.
   $(\forall x\ (RR(x) \rightarrow BEEP(x)) \rightarrow (\forall y\ (COYOTE(y) \rightarrow FRUSTRATED(y)))$

7. Consider the following axioms:

1. Anyone who rides any Harley is a rough character.
   $\forall x\ ((\exists y\ (HARLEY(y) \wedge RIDES(x,y))) \rightarrow ROUGH(x))$
2. Every biker rides [something that is] either a Harley or a BMW.
   $\forall x\ (BIKER(x) \rightarrow \exists y\ ((HARLEY(y) \vee BMW(y)) \wedge RIDES(x,y)))$
3. Anyone who rides any BMW is a yuppie.
   $\forall x\ \forall y\ (RIDES(x,y) \wedge BMW(y) \rightarrow YUPPIE(x))$
4. Every yuppie is a lawyer.
   $\forall x\ (YUPPIE(x) \rightarrow LAWYER(x))$
5. Any nice girl does not date anyone who is a rough character.
   $\forall x\ \forall y\ (NICE(x) \wedge ROUGH(y) \rightarrow \neg\ DATE(x,y))$
6. Mary is a nice girl, and John is a biker.
   $NICE(Mary) \wedge BIKER(John)$
7. (Conclusion) If John is not a lawyer, then Mary does not date John.
   $\neg\ LAWYER(John) \rightarrow \neg\ DATE(Mary,John)$

8. Consider the following axioms:

1. Every child loves anyone who gives the child any present.
   $\forall x\ \forall y\ \forall z\ (CHILD(x) \wedge PRESENT(y) \wedge GIVE(z,y,x) \rightarrow LOVES(x,z)$
2. Every child will be given some present by Santa if Santa can travel on Christmas eve.
   $TRAVEL(Santa,Christmas) \rightarrow \forall x\ (CHILD(x) \rightarrow \exists y\ (PRESENT(y) \wedge GIVE(Santa,y,x)))$
3. It is foggy on Christmas eve.
   $FOGGY(Christmas)$
4. Anytime it is foggy, anyone can travel if he has some source of light.
   $\forall x\ \forall t\ (FOGGY(t) \rightarrow (\ \exists y\ (LIGHT(y) \wedge HAS(x,y)) \rightarrow TRAVEL(x,t)))$
5. Any reindeer with a red nose is a source of light.
   $\forall x\ (RNR(x) \rightarrow LIGHT(x))$
6. (Conclusion) If Santa has some reindeer with a red nose, then every child loves Santa.
   $(\ \exists x\ (RNR(x) \wedge HAS(Santa,x))) \rightarrow \forall y\ (CHILD(y) \rightarrow LOVES(y,Santa))$

9. Consider the following axioms:

1. Every investor bought [something that is] stocks or bonds.
   $\forall x\ (INVESTOR(x) \rightarrow \exists y\ ((STOCK(y) \vee BOND(y)) \wedge BUY(x,y)))$
2. If the Dow-Jones Average crashes, then all stocks that are not gold stocks fall.
   $DJCRASH \rightarrow \forall x\ ((STOCK(x) \wedge \neg\ GOLD(x)) \rightarrow FALL(x))$
3. If the T-Bill interest rate rises, then all bonds fall.
   $TBRISE \rightarrow \forall x\ (BOND(x) \rightarrow FALL(x))$

4. Every investor who bought something that falls is not happy.
   $\forall x\ \forall y\ (INVESTOR(x) \land BUY(x,y) \land FALL(y)\ \text{\&rarrm;}\ \neg\ HAPPY(x))$
5. (Conclusion) If the Dow-Jones Average crashes and the T-Bill interest rate rises, then any investor who is happy bought some gold stock.
   $(\ DJCRASH \land TBRISE\ ) \rightarrow \forall x\ (INVESTOR(x) \land HAPPY(x) \rightarrow \exists y\ (GOLD(y) \land BUY(x,y)))$

10. Consider the following axioms:

1. Every child loves every candy.
   $\forall x\ \forall y\ (CHILD(x) \land CANDY(y) \rightarrow LOVES(x,y))$
2. Anyone who loves some candy is not a nutrition fanatic.
   $\forall x\ (\ (\exists y\ (CANDY(y) \land LOVES(x,y))) \rightarrow \neg\ FANATIC(x))$
3. Anyone who eats any pumpkin is a nutrition fanatic.
   $\forall x\ ((\exists y\ (PUMPKIN(y) \land EAT(x,y))) \rightarrow FANATIC(x))$
4. Anyone who buys any pumpkin either carves it or eats it.
   $\forall x\ \forall y\ (PUMPKIN(y) \land BUY(x,y) \rightarrow CARVE(x,y)\ \lor EAT(x,y))$
5. John buys a pumpkin.
   $\exists x\ (PUMPKIN(x) \land BUY(John,x))$
6. Lifesavers is a candy.
   $CANDY(Lifesavers)$
7. (Conclusion) If John is a child, then John carves some pumpkin.
   $CHILD(John) \rightarrow \exists x\ (PUMPKIN(x) \land CARVE(John,x))$