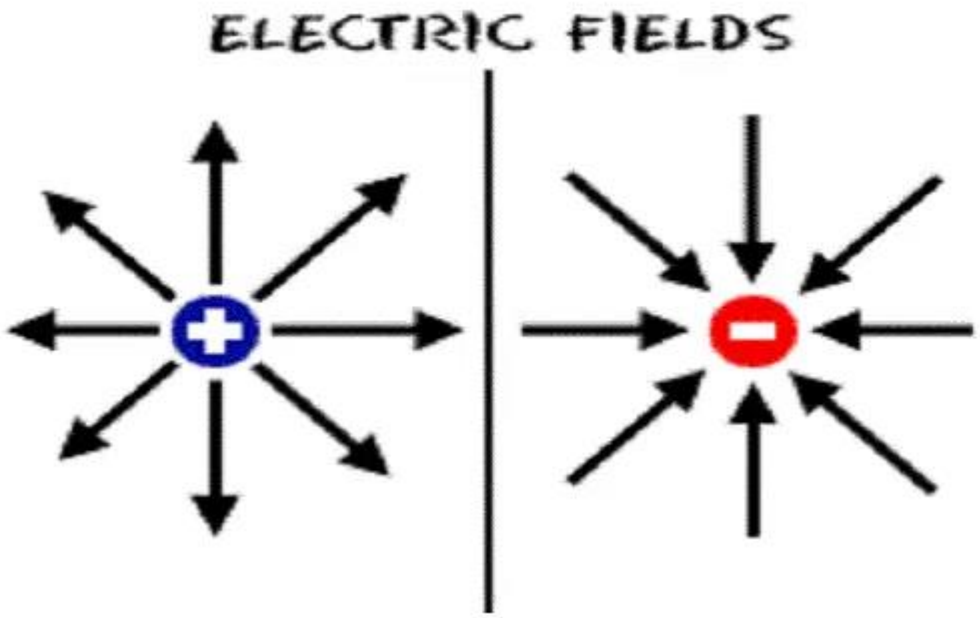
Electric Field due to Point and Continuous Charges

Electric Field

Electrical fields (sometimes referred to as E-fields) refer to the physical field that surrounds electrically charged particles and exert forces on all other charged particles within the field, either attracting or repelling them.

- The electric field between two charges is similar to the **gravitational field** between two **masses** because both of them obey the **inverse-square law** with distance.
- The electric field can be visualized with a set of imaginary lines known as the electric field lines or lines of force.
- **Electric field lines** are originated from a positive charge and terminated on a negative charge.



Electric Charge

Electric charge is the property of a particle due to which it can apply force on another charged or uncharged particle.

- It is a scalar quantity and the SI unit is Coulomb (C).
- Dimensional formula of electric charge is [AT].
- The charge associated with a proton is known as a positive charge.

- The charge associated with an electron is known as a negative charge.

Electric Field of a Point Charge Formula

The electric field intensity (E) due to a point charge (Q) at any point in its electric field is defined as the electrostatic or Coulomb's force (F) per unit of charge exerted on an infinitesimal positive test charge (q_o) at rest at that point.

Mathematically, the electric field intensity is given by

$$E = F/q_o$$

- It is a vector quantity.
- SI unit of electric field intensity is Newton per coulomb (NC^{-1}) or Volt per meter (Vm^{-1}).
- Dimensional formula of electric field intensity is $[MLT^{-3}A^{-1}]$

Electric field due to a point charge

According to Coulomb's law, the force between two charges Q and q_o separated by distance r is given by

$$F = \frac{1}{4\pi\epsilon_o}\frac{Qq_o}{r^2}$$

Where ϵ_o is the absolute permittivity of free space.

Therefore, the electric field intensity is given by

$$E = \frac{F}{q_o} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}$$

Above expression represents the electric field due to a point at distance r.

Derivation of Electric Field Due to Two Point Charges

Consider two point charges q_1 & q_2 placed at A and B having position vector

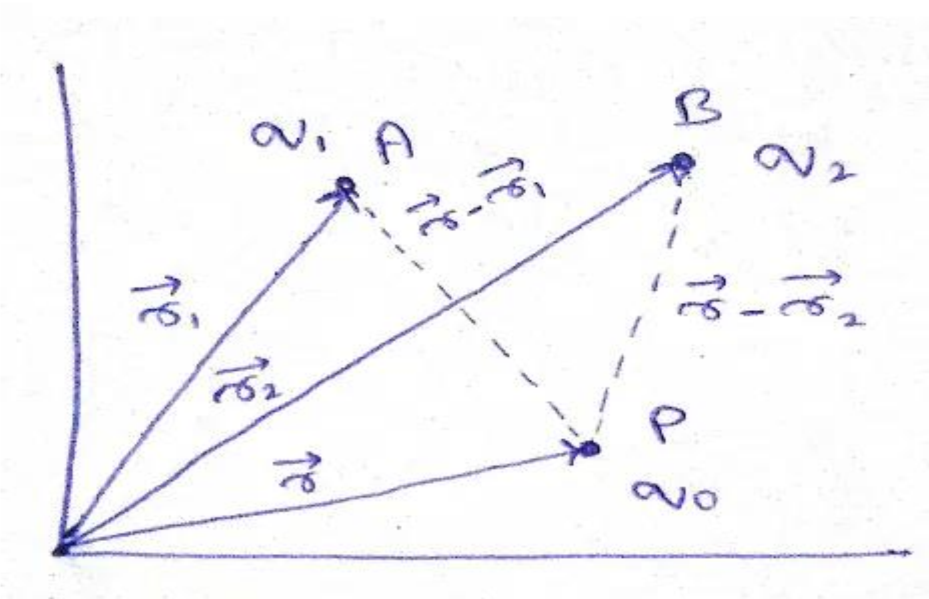
$\vec{r_1}$ and $\vec{r_2}$

The force on q_o due to q_1 is given by

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1})$$

Force on q_o due to q_2 is given by

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_2}{|\vec{r}-\vec{r_2}|^3}(\vec{r}-\vec{r_2})$$



Electric field lines

Net force acting on q_o placed at point P is given by

$F = F_1 + F_2$

$$\Rightarrow F = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1}) + \frac{1}{4\pi\epsilon_0}\frac{q_0 q_2}{|\vec{r}-\vec{r_2}|^3}(\vec{r}-\vec{r_2})$$

$$\Rightarrow F = \frac{q_0}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1}) + \frac{q_2}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_2})\right]$$

Electric field intensity at point P, is given by

$$\vec{E} = \frac{\vec{F}}{q_o}$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_1}) + \frac{q_2}{|\vec{r}-\vec{r_1}|^3}(\vec{r}-\vec{r_2})\right]$$

Electric Field Due to a System or Group of Point Charges

The electric field intensity at any point because of a system or group of charges is equivalent to the vector sum of electric field intensities because of individual charges at the same point. Currently, we would perform the vector sum of electric field intensities:

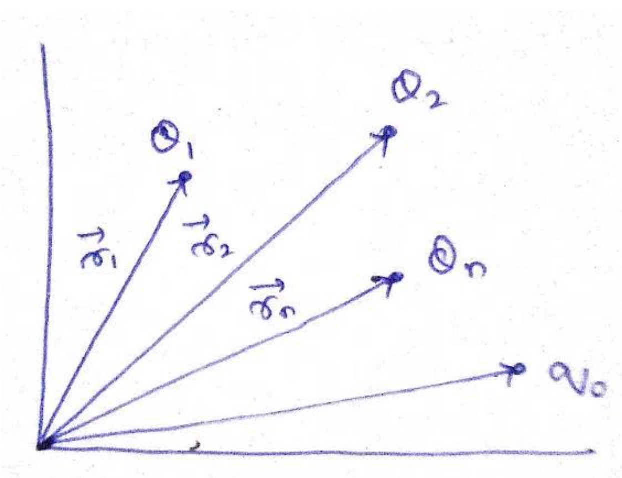
$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \ldots + \vec{E_n}$$

$$\vec{E_1} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{i=n}\frac{Q_i}{r_i^2}(\hat{ri})$$

Here,

- r_i is the distance of the point P from the i^{th} charge Q_i
- r_i is a unit vector directed to the point P from Q_i.

Let's say charge Q_1, Q_2...Q_n is positioned in a vacuum at places r_1, r_2...., and $r_?$ respectively.



Electric field due to multiple point charges

The net forces at P are the vector sum of forces because of individual charges, given by,

$$\vec{F} = \frac{1}{4\pi\epsilon_0} q_0 \sum_{i=1}^{i=n} \frac{Q_i}{|\vec{r}-\vec{ri}|^3 \times |\vec{r}-\vec{ri}|}$$

As, $\vec{E} = \frac{\vec{F}}{q_o}$

Therefore,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{i=n} \frac{Q_i}{|\vec{r}-\vec{ri}|^3 \times |\vec{r}-\vec{ri}|}$$

Putting $\frac{1}{4\pi\epsilon_0}$ = k

$$\vec{E} = k\frac{Q_1}{r_1^2} + k\frac{Q_2}{r_2^2} + \ldots\ldots\ldots\ldots + k\frac{Q_n}{r_n^2}$$

Electric Field due to Continous Distribution of Charge

A system of closed spaced electric charges forms a continuous charge distribution. It can be on a line, on a surface, or on the entire volume.

To calculate the electric field due to charge distribution, the concept of charge density is introduced.

1. **Linear charge density (λ):** It is defined as the charge per unit length of the conductor. It is given by

$$\lambda = q/l$$

2. **Surface charge density (σ):** It is defined as the charge per unit area of the conductor. It is given by

$$\sigma = q/A$$

3. **Volume charge density (ρ):** It is defined as the charge per unit volume of the conductor. It is given by

$$\rho = q/V$$

Electric field intensity due to Linear charge distribution is given by

$$E = \frac{F}{q_o} = \frac{1}{4\pi\epsilon_o} \int_L \frac{\lambda dL}{r^2}$$

Electric field intensity due to Surface charge distribution is given by

$$E = \frac{F}{q_o} = \frac{1}{4\pi\epsilon_o} \int_A \frac{\sigma dA}{r^2}$$

Electric field intensity due to Volume charge distribution is given by

$$E = \frac{F}{q_o} = \frac{1}{4\pi\epsilon_o} \int_V \frac{\rho dV}{r^2}$$