

(An Autonomous Institution) Coimbatore-641035.



UNIT 4– ALGEBRAIC STRUCTURES



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Inverse of i is
$$-1$$
 is, $1+-1=1 \in G_1$
Inverse $O_0 = 1$ is $1 \otimes g_1 = 1 = 1 \in G_1$
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Groups

V). commutative: 1 * 0 = 00 & A W * 1 = WE A Hence (A, *) is an abelgan goloup. 3]. Let I be the set of integers. Let Zon be the Bet of equilivalence classes generated by the equavalence relation " congraence modulo m' tor any the Portages in. There (Zm, tm) and (Zm, Ym) are monords. Soln. FOR [1], [j] E Im a). +m is defined as [i]+m[j]=[i+j)(mod m)] b). Xm & defend as [i] xm[j]=[(1xj) (mod m)] The composition table for m= 5 is given as $(Z_{5}, +_{5})$ (Z_{E}, Y_{E}) + 0 1 2 3 4 XEOIQ 3 4 0 0 10123 4 2340 1 2 2 0 2 4 1 3 3401 2 3 0 3 1 4 3 B 0 4 3 2 1 40123 4 4 1). closure proporty: In the above table (Z5, t5) and (Z5, X5 Sat7870s closure property. ii) Assocrative: clearly, (75, 75) and (75, ×5) satisfies associative proporty. 11), Identify ett. : IOJ 23 the identifier elt. w.r. to to LIT 33 the



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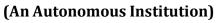


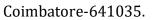
UNIT 4– ALGEBRAIC STRUCTURES

$$(7z_{m}, +m) \text{ and } (7z_{m}, x_{m}) \text{ are monords.}$$

$$HJ. Show that $(a^{+}, *)$ is an apellian group
where $*$ is defined by $a * b = \frac{ab}{2}$, $* c_{ab} \in a^{+}$
is a^{+} is $dosed$
i). For $a, b \in a^{+} \Rightarrow a * b = \frac{ab}{2} \in a^{+}$
 $\therefore a^{+}$ is $dosed$
ii). For $a, b, c \in a^{+}$. Then $a * (b * c) = a + \frac{bc}{2}$
 $= \frac{abc}{4} \Rightarrow (a)$
 $a * (b * c) = a + \frac{bc}{2}$
 $= \frac{abc}{4} \Rightarrow (a)$
 $a * (b * c) = (a * b) * c$
iii) I fdentify:
 $1 \text{ bet } a \in a^{+}$. Then $f \in e \in a^{+}$ such that
 $Now \quad a * e = \frac{a}{2}$
 $\frac{ae}{2} = a \Rightarrow e = a$
iv). Invertise eff.:
 $1 \text{ bet } a \in a^{+}$. Then $f a^{+} e = a$
 $a = \frac{aa^{-}}{2} = e$
 $a = \frac{aa^{-}}{2} = e$
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F]. Let Gi denote the set of all matifies of the
point
$$\begin{bmatrix} x & x \\ y & x \end{bmatrix}$$
 where $x \in R$. Prove that Gi is a
group evolution matifies mentification.
Solo.
U closate:
Let $R, B \in Gi$
Let $R = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$; $B = \begin{bmatrix} y & y \\ y & y \end{bmatrix}$
Then $RB = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \begin{bmatrix} y & y \\ y & y \end{bmatrix} = \begin{bmatrix} xy + xy & xy + xy \\ xy + xy & xy + xy \end{bmatrix}$
 $= \begin{bmatrix} 2xy & 2xy \\ axy & 2xy \end{bmatrix} \in G_1$
i)). Associative:
Nations multipleation is Associative.
Nations multipleation is Associative.
Nations multipleation is Associative.
Now, $\begin{bmatrix} x & x \\ x & x \end{bmatrix} \begin{bmatrix} d & e \end{bmatrix} = \begin{bmatrix} x & x \\ x \end{bmatrix}$
 $pxe = x \Rightarrow e = y_2$
Hence $E = \begin{bmatrix} y_2 & y_2 \\ x & x \end{bmatrix} \begin{bmatrix} z & x \\ e & x \end{bmatrix}$ is the denotify eit. of G.
iv). Therese ett:
Let $A = \begin{bmatrix} x & x \\ x \end{bmatrix}$ is the denotify eit. of G.
iv). Threese ett:
Let $A = \begin{bmatrix} x & x \\ x \end{bmatrix}$. Then $F = A^{-1} = \begin{bmatrix} x & x \\ x \end{bmatrix} =$
 $AA^{-1} = E \Rightarrow \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} y & y \\ e \end{bmatrix} = \begin{bmatrix} y_2 & y_2 \\ x \end{bmatrix}$
 $e^{2xx} & e^{2xe} \end{bmatrix} = \begin{bmatrix} x & x \\ x \end{bmatrix}$
 $AA^{-1} = E \Rightarrow \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} y & y \\ e \end{bmatrix} = \begin{bmatrix} y_2 & y_2 \\ x \end{bmatrix}$
 $e^{2xx} & e^{2xe} \end{bmatrix} = \begin{bmatrix} x & x \\ x \end{bmatrix}$
 $e^{2xx} & e^{2xe} \end{bmatrix} = \begin{bmatrix} x & x \\ x \end{bmatrix}$
 $e^{2xx} & e^{2xe} \end{bmatrix} = \begin{bmatrix} x & x \\ x \end{bmatrix}$
 $AA^{-1} = E \Rightarrow \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} x & x \\ x \end{bmatrix} \begin{bmatrix} y & y \\ e \end{bmatrix} = \begin{bmatrix} y & y \\ x \end{bmatrix}$
 $e^{2xx} & e^{2xe} \end{bmatrix} = \begin{bmatrix} x & x \\ x \end{bmatrix}$
 $e^{2xx} & e^{2xe} \end{bmatrix} = \begin{bmatrix} x & x \\ y \end{bmatrix}$
Hence $A^{-1} = \begin{bmatrix} x & x \\ x \\ x \end{bmatrix} \begin{bmatrix} x & x \\ x \end{bmatrix} = \begin{bmatrix} y & y \\ y \end{bmatrix}$







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