SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution) COIMBATORE – 35

23MAT201 –PARTIAL DIFFERENTIAL EQUATIONS AND TRANSFORMS

UNIT-III

APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART-B

1. Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$
2. Obtain the solution of one-dimensional wave equation.
3. A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t.
4. A string of length 2l is fastened at both ends. The mid-point of the string is taken to a height b and then released from rest in that position. Show that the displacement is $y(x,t) = \frac{8b}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^2}\sin\left(\frac{(2n-1)\pi x}{2l}\right)\cos\left(\frac{(2n-1)\pi at}{2l}\right)$
5. A Tightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$. The mid-point of the string is then taken to a height h and then released from the rest in that position. Obtain an expression for displacement of the string at any subsequent time.
6. A Tightly stretched string with fixed end points $x = 0\ and\ x = l$ is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity $\lambda x(l - x)$ show that the displacement is $y(x,t) = \frac{8\lambda l^3}{a\pi^4}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}\sin\left(\frac{(2n-1)\pi x}{l}\right)\cos\left(\frac{(2n-1)\pi at}{l}\right)$
7. If a string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $v_0 sin^3\frac{\pi x}{l}$, $0 < x < l$, determine the displacement of a point distant x from one end at time 't'.
8. A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity given by

$$v = \begin{cases} cx & if\ 0 \le x \le \frac{l}{2} \\ c(l-x) & if\ \frac{l}{2} \le x \le l \end{cases}$$ and find the displacement function $y(x,t)$.

9. Obtain the solution of Heat equation.
10. What is the assumption made while deriving one dimensional heat equation?
11. What is the basic difference between the solution of one-dimensional wave equation and one-dimensional heat equation?
12. A rod 30cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail. The temperature at each is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function $u(x,t)$ taking $x = 0$ at A.
13. The ends A and B of a rod 30cms long have their temperature kept at 20^0C and the other at 80^0C until steady state conditions prevail. The temperature of the end B is then suddenly reduced to 60^0C and kept so while the end A is raised to 40^0C. Find the temperature distribution in the rod after time t.
14. A rectangular plate is bounded by the lines $x = 0, y = 0, x = a$ and $y = b$. Its surface is insulated and the temperature along two adjacent edges are kept at 100^0C, while the temperature along the other two edges is at 0^0C. Find the steady state temperature at any point in the plate. Also find the steady state temperature at any point of a square plate of side 'a' if two adjacent edges are kept at 100^0C and the others at 0^0C.
15. A square plate is bounded by the lines $x = 0, y = 0, x = l, y = l$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x,l) = x(l-x)$ while the other three edges are kept at 0^0C. Find the steady state temperature in the plate.
16. Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions
    $u(0,y) = u(a,y) = 0$ for $0 \le y \le b$
    $u(x,b) = 0$ and $u(x,0) = x(a-x)$ for $0 \le x \le a$