UNIT 5 Z - Transforms and Difference equations
Initial and Final value Theorem

Initial Value Theorem:-

If $z[f(t)] = F(z)$, then $f(0) = \lim_{z\to\infty} F(z)$

Final Value Theorem:

If $z[f(t)] = F(z)$, then $\lim_{t\to\infty} f(t) = \lim_{z\to 1} (z-1)F(z)$

Problems on IVT & FVT

1. If $F(z) = \frac{5z}{(z-2)(z-3)}$, find $f(0)$ and $\lim_{t\to\infty} f(t)$

By IVT

$$f(0) = \lim_{z\to\infty} F(z) = \lim_{z\to\infty} \frac{5z}{(z-2)(z-3)} = \lim_{z\to\infty} \frac{5z}{z^2-5z+6}$$

$$= \lim_{z\to\infty} \frac{5}{2z-5} \quad \text{[By L Hôpital's Rule]}$$

$$= 0.$$

By FVT,

$$\lim_{t\to\infty} f(t) = \lim_{z\to 1} (z-1)F(z) = \lim_{z\to 1} (z-1)\frac{5z}{(z-2)(z-3)} = 0.$$

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2. Find the Initial & Final Value of $F(z) = \frac{z}{2z^2-3z+1}$

By IVT, $\lim_{z\to\infty} F(z) = f(0)$

$$f(0) = \lim_{z\to\infty} \frac{z}{2z^2-3z+1} = \lim_{z\to\infty} \frac{1}{4z-3}$$

$$= 0.$$

By FVT, $\lim_{t\to\infty} f(t) = \lim_{z\to1} (z-1)F(z)$

$$= \lim_{z\to1} (z-1)\frac{z}{2z^2-3z+1}$$

$$= \lim_{z\to1} (z-1)\frac{z}{(2z-1)(z-1)}$$

$$= \lim_{z\to1} \frac{z}{2z-1}$$

$$= \frac{1}{2(1)-1} = \frac{1}{1} = 1$$