



UNIT4-ALGEBRAIC STRUCTURES

Semigroups and Monoids

Notations :

- \mathbb{Z} \rightarrow the set of all integers
- \mathbb{Q} \rightarrow the set of all rational numbers
- \mathbb{R} \rightarrow the set of all real numbers
- \mathbb{R}^+ \rightarrow the set of all positive real numbers
- \mathbb{Q}^+ \rightarrow the set of all positive rational numbers
- \mathbb{C} \rightarrow the set of all complex numbers.

Semigroup :

If a non empty set S together with the binary operation $*$ satisfying the following properties.

- i). closure property
- ii). Associative property, is called semigroup

Monoid :

A non empty set M together with the binary operation $*$ satisfying the following properties.

- i). closure
- ii). Associative
- iii). Identity, is called monoid.

Examples :

- 1]. $(\mathbb{Z}, \rightarrow)$ is the example of quasi group. (closure)
- 2]. $(\mathbb{N}, +) \rightarrow$ semigroup [closure & Associative]
- 3]. since '0' is not a natural no., here '0' is the identity.
- 3]. $(\mathbb{N}, *) \rightarrow$ monoid [closure, Associative & Identity]
- since fraction does not exist in natural no.
- 4]. $(\mathbb{Z}, +) \rightarrow$ Group & Abelian group [closure, Associative, Identity, Inverse & Commutative]

Order of a group :

The number of elements in a group G is called the order of a group & is denoted by $O(G)$

- i). If $O(G)$ is finite, then G is said to be finite group.
- ii). If $O(G)$ is infinite, then G is said to be infinite group.