Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 40784

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Computer Science and Engineering

MA 8351 – DISCRETE MATHEMATICS

(Common to Artificial Intelligence and Data Science/Computer Science and Business System/Information Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. When do you say that two compound propositions are equivalent?
2. Define Tautology with an example.
3. Does there exists a simple graph with the degree sequence $\{3, 3, 3, 3, 2\}$?
4. State the Pigeonhole principle.
5. Define strongly connected graph.
6. Define complete graph.
7. Prove if a has inverse b and b has inverse c, then $a = c$.
8. Prove that identity element is unique in a group.
9. Define lattice homomorphism.
10. When is a lattice said to be a Boolean algebra?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) When do we say a formula is tautology or contradiction? Without constructing truth table, verify whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a contradiction or tautology. Justify your answer. (6)

(ii) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. (10)

Or

(b) (i) Let m and n be integers. Prove that $n^2 = m^2$ if and only if $m = n$ or $m = -n$.

(ii) Write down the negation of each of the following statements :

(1) For all integers n, if n is not divisible by 2, then n is odd

(2) If k, m, n are any integers, where $(k-m)$ and $(m-n)$ are odd, then $(k-n)$ is even.

(3) For all real numbers x, if $|x-3|<7$, then $-4<x<10$.

(4) If x is real number where $x^2 > 16$, then $x < -4$ or $x > 4$.

12. (a) (i) Use mathematical induction to show that $2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 1$, for all non-negative integers n. (8)

(ii) State the Inclusion and Exclusion principle. Hence, using the principle, find how many faculty members can speak either French or Russian, if 200 faculty members can speak French and 50 can speak Russian, while only 20 can speak both French and Russian. (8)

Or

(b) (i) Solve the recurrence relation, $S(n) = S(n-1) + 2(n-1)$ with $S(0) = 3$, $S(1) = 1$ by finding its generating function.

(ii) Prove by mathematical induction that for every positive integer n, 3 divides $n^3 - n$.

13. (a) (i) Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete subgroup of K_5 with 4 vertices.

(ii) If $(S_1, *)$ and $(S_2, \circ)$ are two semigroups such that $f : S_1 \to S_2$ is an onto homomorphism and a relation R is defined on S, Such that $aRb \Leftrightarrow f(a) = f(b)$ for any $a, b \in S_1$ then R is a congruence relation.

Or

(b) (i) Define :

(1) Adjacency matrix and

(2) Incidence matrix of a group with examples.

(ii) Prove that any undirected graph has an even number of vertices of odd degree.

14. (a) (i) Prove that the group homomorphism preserves the identity element.

(ii) Let f be a group homomorphism from a group $(G, *)$ into a group (H, Δ) then prove that $\ker(f)$ is a subgroup. Check whether $\ker(f)$ is a normal subgroup of $(G, *)$. Justify your answer. (8)

Or

(b) (i) State and prove Lagrange's theorem.

(ii) Show that the union of two subgroups of a group G is subgroup of G if and only if one is contained in other.

15. (a) (i) Let $(L, \leq)$ be a lattice. For any $a, b \in L$, $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$.

(ii) Draw the lattice of (S, gcd, lcm) where $S = \{x : x$ is a divisor of 210$\}$.

Or

(b) (i) Show that $(N, \leq)$ is a partially ordered set where N is set of all positive integers and $\leq$ is defined by $m \leq n$ iff $n - m$ is a non-negative integer.

(ii) Prove that $(L, \wedge, \vee)$ is not a complemented lattice (under division relation) where $L = \{1, 2, 3, 4, 6, 12\}$ and also draw the Hasse diagram.

Roll No. | | | | | | | | | | |

QUESTION PAPER CODE: X10657

B.E. / B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Third Semester
Computer Science and Engineering
MA8351 –DISCRETE MATHEMATICS
(Common to Information Technology)
(Regulations 2017)
Answer ALL Questions

Time: 3 Hours **Maximum Marks:100**

PART-A

(10×2=20 Marks)

1. Show that $\{\neg, \wedge\}$ is a functionally complete set of connectives.

2. Write the negation of the statement $\forall(x^2 > x) \wedge \exists x(x^2 = 4)$.

3. Using the principle of mathematical induction, show that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$, $\forall \quad n \geq 1$.

4. In how many ways a foot ball team of eleven players can be chosen out of 17 players, when

   (i) five particular players are to be always included.

   (ii) two particular players are to be always exclued.

5. Obtain the adjacency matrix of the complement of the graph $K_{1,4}$.

6. Check whether the complete bipartite graph $K_{3,3}$ is Hamiltonian or Eulerian.

7. In a group $(G, *)$, show that $(a * b)^{-1} = b^{-1} * a^{-1}$, $\forall\, a, b \in G$.

8. Show that if every element of group is self-inverse then it must be abelian.

9. Show that in a partially ordered set $(A, \leq)$, if greatest lower bound of a subset $S \subseteq A$ exists, then it must be unique.

10. In a lattice $(L, \leq)$, show that $a \leq b$, if and only if $a * b = a$.

PART-B

(5×16=80 Marks)

11. (a) (i) Use the indirect method to show that

$$R \to \neg Q, \quad R \cup S, \quad S \to \neg Q, \quad P \to Q \implies \neg P$$

(8)

(ii) Show that the premises "A student in the class has not read the book" and "Every one in this class passed the semester exam" imply the conclusion "Some one who passed the semester exam "has not read the book". (8)

(OR)

(b) (i) Using indirect method, prove the following statements.

(A) If n is an integer and $3n + 2$ is odd, then n is odd. (4

(B) If $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$. (4

(ii) Construct an argument to show that the following premises to show that the following premises imply the conclusion "It rained". "If it does not rain or if there is no traffic dislocation, then the soports day will be held and the cultural programme will go on"; "If the sports day is held, the trophy will be awarded" and "The trophy was not awarded". (8)

12. (a) (i) Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$, where $n \geq 2$ and $a_0 = 1$, $a_1 = 2$. (10)

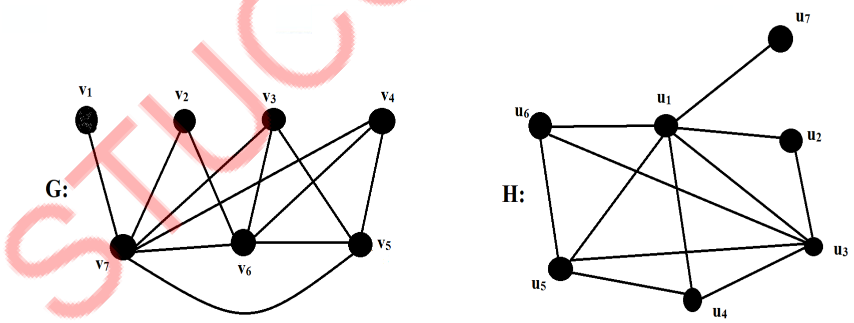
(ii) Prove that every positive integer $n \geq 2$ is either a prime or it is a product of primes. (6)

(OR)

(b) (i) Determine the number of positive integers n, $1 \leq n \leq 2000$ that are not divisible by 2, 3, or 5 but are divisible by 7. (10)

(ii) An odd positive integer n such that m denotes $2^n - 1$. (6)

13. (a) (i) State the necessary condition for two graphs to be isomorphic. Show that the following two graphs are isomorphic. (10)



(ii) State and prove Hand-Shake lemma for graphs. (6)

(OR)

(b) (i) When do we say a graph is self-complementary. If a graph G is self-complementary then prove that $|V(G)| \equiv 0 \ \ 1 \ (\text{mod} 4)$ (6)

(ii) Let G be a graph with $S(G) \geq \frac{|V(G)|}{2}$ and $|V(G)| \geq 3$. Then prove that G is Hamiltonian. (6)

14. (a) (i) IF $(G, *)$ is a finite group, then prove that order of any subgroup divides the order of the group. (10)

(ii) Prove that group homomorphism preserves identity and inverse. (6)

(OR)

(b) (i) Obtain the composition table of $(S_3, \Diamond)$ and show that $(S_3, \Diamond)$ is a group/ Check whether $(S_3, \Diamond)$ is abelian. Justify your answer. (10) (8)

(ii) Show that in a cycle group every subgroup is a normal subgroup. (6)

15. (a) (i) Lat (A, R) be a partially ordered set. Then show that $A, R^{-1})$ is also partially set, where R_{-1} is defined as $R^{1-} = \{(a, b) \in A \times A/(b, a) \in R\}$. (6)

(ii) Show that in a lattice "isotone property" and "distributive inequalities" are true. (10)

(OR)

(b) (i) Show that in a distributive lattice cancellation law is true. Hence, show that in a distributive lattice if compliment of an element exists then it must be unique. (6)

(ii) Show that the complemented and distributive lattice, the following are true.

$$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$$

(10)

* * * * * * *

15. (a) (i) If (A, R) is a partially ordered set then show that the set (A, R^{-1}) is also a partially ordered set, where $R^{-1} = \{(b, a) / (a, b) \in R\}$. (6)

(ii) Let $(L, *, \oplus)$ and $(M, \wedge, \vee)$ be two lattices. Then prove that $(L \times M, \Delta, \nabla)$ is a lattice, where $(x, y)\Delta(a, b) = (x * a, y \wedge b)$ and $(x, y)\nabla(a, b) = (x \oplus a, y \vee b)$, for all $(x, y), (a, b) \in L \times M$. (10)

Or

(b) (i) Prove that in every lattice distributive inequalities are true. (8)

(ii) Define modular lattice. Prove that a lattice L is modular if and only if $x, y \in L$, $x \oplus (y * (x \oplus z)) = (x \oplus y) * (x \oplus z)$. (8)

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Reg. No. :

Question Paper Code : 80212

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

Computer Science and Engineering

MA 8351 — DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulation 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the inverse of the statement, "If you work hard then you will be rewarded".

2. If the universe of discourse consists of all real numbers and if $p(x)$ and $q(x)$ are given by $p(x): x \geq 0$ and $q(x): x^2 \geq 0$, then determine the truth value of $(\forall x)(p(x) \to q(x))$.

3. Prove that if n and k are positive integers with $n = 2k$, then $\frac{n!}{2^k}$ is an integer.

4. How many solutions does the equation, $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are non-negative integers?

5. If G is a simple graph with $\delta(G) \geq \frac{|V(G)|}{2}$ then show that G is connected.

6. Give an example of a graph which is Hamiltonian but not Eulerian.

7. Is it true that $(\mathbb{Z}_5^*, \times_5)$ a cyclic group? Justify your answer.

8. Prove that group homomorphism preserves identity.

9. Show that in a lattice if $a \le b$ and $c \le d$ then $a * c \le b * d$.

10. Is it true that every chain with at least three elements is always a complemented lattice? Justify your answer.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal conjective normal form of the formula $(\urcorner P \to R) \wedge (P \to Q) \wedge (Q \to P)$. (6)

(ii) Using indirect method, show that $R \to \urcorner Q$, $R \vee S$, $S \to \urcorner Q$, $P \to Q \Rightarrow \urcorner P$. (10)

Or

(b) (i) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the Semester Exam" imply the conclusion "Someone who passed the Semester Exam has not read the book". (10)

(ii) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. (6)

12. (a) (i) Let $m \in Z^+$ with m odd. Then prove that there exists a positive integer n such that m divides $2^n - 1$. (6)

(ii) Determine the number of positive integers n, $1 \le n \le 2000$ that are not divisible by 2, 3 or 5, but are divisible by 7. (10)

Or

(b) (i) Using mathematical induction, prove that every integer $n \ge 2$ is either a prime number or product of prime numbers. (6)

(ii) Using generating function method solve the recurrence relation, $a_{n+2} - 2a_{n+1} + a_n = 2^n$, where $n \ge 0$, $a_0 = 2$ and $a_1 = 1$. (10)

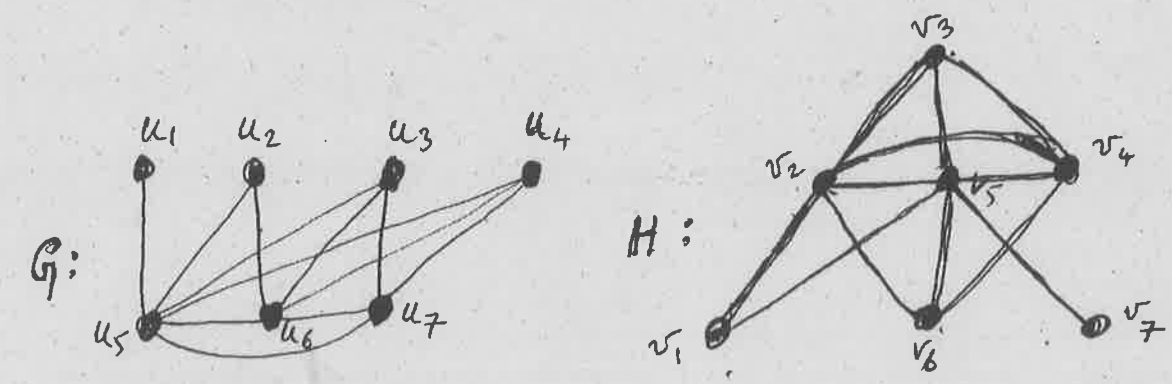
13. (a) (i) Let G be a graph with adjacency matrix A with respect to the ordering of vertices $v_1, v_2, v_3, \ldots, v_n$. Then prove that the number of different walks of length r from v_i to v_j, where r is a positive integer, equals to $(i, j)^{th}$ entry of A^r. (8)

(ii) Show that the complete bipartite graph $K_{m,n}$, with $m, n \ge 2$ is Hamiltonian if and only if $m = n$. Also show that the complete graph K_n is Hamiltonian for all $n \ge 3$. (8)

Or

(b) (i) Define incidence matrix of a graph. Using the incidence matrix of a graph G, show that the sum of the degrees of vertices of a graph G is equal to twice the number of edges of G. (6)

(ii) When do we say two simple graphs are isomorphic? Check whether the following two graphs are isomorphic or not. Justify your answer. (10)



14. (a) (i) Prove that every subgroup of a cyclic group is cyclic. (6)

(ii) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (10)

Or

(b) (i) Define monoid. Give an example of a semigroup that is not a monoid. Further prove that for any commutative monoid $(M, *)$, the set of idempotent elements of M form a submonoid. (8)

(ii) Let $(G, *)$ be a group and let H be a normal subgroup of G. If G/H be the set $\{aH \mid a \in G\}$ then show that $(G/H, \otimes)$ is a group, where $aH \otimes bH = (a * b)H$, for all $aH, bH \in G \mid H$. Further, show that there exists a natural homomorphism $f : G \to G \mid H$. (8)

15. (a) (i) State and prove distributive inequalities in lattices. (8)

(ii) Prove that every chain is a distributive lattice. (8)

Or

(b) (i) Consider the set $D_{50} = \{1, 2, 5, 10, 25, 50\}$ and the relation divides (/) be a partial ordering relation on D_{50}. (8)

(1) Draw the Hasse diagram of D_{50} with relation divides.

(2) Determine all upper bounds of 5 and 10.

(3) Determine all lower bounds of 5 and 10.

(4) Determine LUB. of 5 and 10.

(5) Determine GLB. of 5 and 10.

(ii) State and prove De Morgan's laws in complemented and distributive lattice. (8)

22/11/18

Reg. No. :

Question Paper Code : 25139

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Computer Science and Engineering

MA 8351 — DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the truth table for the following $P \wedge (P \vee Q)$.

2. Let $Q(x, y, z)$ denote the statement "$x + y = z$" defined on the universe of discourse Z, the set of all integers. What are the truth values of the propositions $Q(1,1,1)$ and $Q(1,1,2)$.

3. Show that in any group of 8 people at least two have birthdays which falls on same day of the week in any given year.

4. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 0$.

5. An undirected graph G has 16 edges and all the vertices are of degree 2. Find the number of vertices?

6. Define incidence matrix of a simple graph.

7. Prove that in any group, identity element is the only idempotent element.

8. Let $f : (G, *) \to (G', \Delta)$ be a group homomorphism. Then prove that $[f(a)]^{-1} = f(a^{-1}), \forall a \in G$.

9. Define partial ordered set.

10. Determine whether D_8 is a Boolean algebra?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the principle disjunctive normal form (PDNF) of

$(P \wedge Q) \vee (\urcorner P \wedge R) \vee (Q \wedge R)$ without using truth table also find its Principle conjunctive normal form. (8)

(ii) Show that if x and y are integers and both xy and $x+y$ are even, then both x and y are even. (8)

Or

(b) (i) Show that $(P \vee Q) \wedge \urcorner(\urcorner P \wedge (\urcorner Q \vee \urcorner R)) \vee (\urcorner P \wedge \urcorner Q) \vee (\urcorner P \wedge \urcorner R)$ is a tautology without using truth table. (8)

(ii) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first examination" imply the conclusion "Someone who passed the first examination has not read the book". (8)

12. (a) (i) Prove by mathematical induction. (8)

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(ii) Solve the recurrence relations $S(n) = S(n-1) + 2S(n-2)$ with $S(0) = 3, S(1) = 1; n \geq 2$ using generating function. (8)

Or

(b) (i) Find the number of integers between 1 to 100 that are not divisible by any of the integers 2,3,5 or 7. (8)

(ii) How many permutations can be made out of the letters of the word "Basic"? How many of these (8)

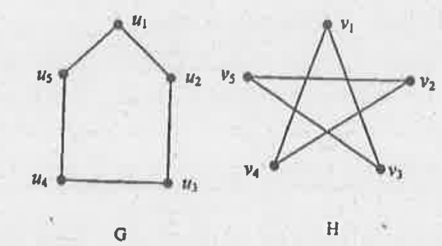
(1) Begin with B?

(2) End with C?

(3) B and C occupy the end places?

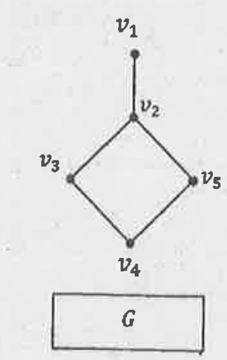
13. (a) (i) Prove that for a bipartite graph with n vertices has maximum of $\frac{n^2}{4}$ edges. (8)

(ii) Establish the isomorphic for the following graphs. (8)



Or

(b) (i) Define a subgraph. Find all the subgraphs of the following graph by deleting an edge. (8)



(ii) Prove that a connected graph has an Euler path if and only if and only if it has exactly two vertices of odd degree. (8)

14. (a) (i) Let $< S, * >$ be a semi group such that for $x, y \in S, x * x = y$, where $S = \{x, y\}$. Then prove that (8)

(1) $x * y = y * x$

(2) $y * y = y$

(ii) Find all the non-trivial subgroups of $(Z_{12}, +_{12})$. (8)

Or

(b) (i) Prove that $G = \{[1],[2],[3],[4]\}$ is an abelian group under multiplication modulo 5. (8)

(ii) Prove that intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

Reg. No. :

Question Paper Code : 90336

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Computer Science and Engineering

MA 8351 – DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulations 2017)

Time : Three Hours　　　　Maximum : 100 Marks

Answer ALL questions

PART – A　　　　**(10×2=20 Marks)**

1. Write the following statement in symbolic form : If Avinash is not in a good mood or he is not busy, then he will go to New Delhi.

2. Write the truth table for $(p \wedge q) \to (p \vee q)$.

3. Find the number of bit strings of length 10 that either begin with 1 or end with 0.

4. In how many different ways can five men and five women sit around a table ?

5. Give an example of a graph which is Eulerian but not Hamiltonian.

6. Write the adjacency matrix and incidence matrix of $K_{2,2}$.

7. Show that the identity element of a group is unique.

8. Give an example of an integral domain which is not a field.

9. Draw the Hasse diagram of $(D_{20}, /)$, where D_{20} denotes the set of positive divisors of 20 and / is the relation "division".

10. In any lattice $(L, \leq)$, $\forall$ a, b $\in$ L, show that $a * (a \oplus b) = a$, where $a * b = glb(a, b)$ and $a \oplus b = lub(a, b)$.

PART – B (5×16=80 Marks)

11. a) i) Obtain the principal disjunctive and conjunctive normal forms of the formula $(\sim p \to r) \wedge (q \leftrightarrow p)$. (8)

ii) Show that $J \wedge S$ logically follows from the premises $P \to Q$, $Q \to \sim R$, R, $P \vee (J \wedge S)$. (8)

(OR)

b) i) Let K(x) : x is a two-wheeler, L(x) : x is a scooter, M(x) : x is manufactured by Bajaj. Express the following using quantifiers.

I. Every two wheeler is a scooter.

II. There is a two-wheeler that is not manufactured by Bajaj.

III. There is a two-wheeler manufactured by Bajaj that is not a scooter.

IV. Every two-wheeler that is a scooter is manufactured by Bajaj. (8)

ii) Use the rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on", "If the sailing race is held, then the trophy will be awarded", and "The trophy was not awarded" imply the conclusion "It rained". (8)

12. a) i) Solve $a_n = 8a_{n-1} + 10^{n-1}$ with $a_0 = 1$ and $a_1 = 9$ using generating function. (8)

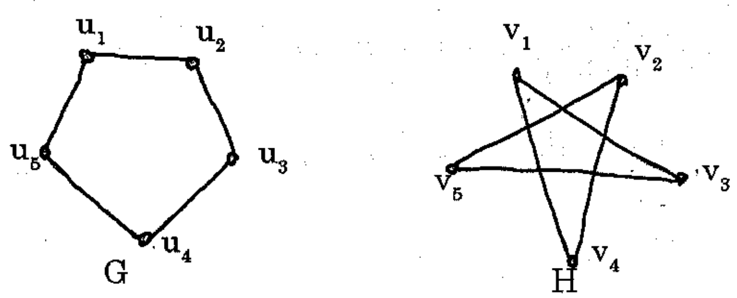
ii) How many positive integers not exceeding 1000 are divisible by none of 3, 7 and 11 ? (8)

(OR)

b) i) Using mathematical induction prove that if n is a positive integer, then 133 divides $11^{n+1} + 12^{2n-1}$. (8)

ii) How many ways are there to assign five different jobs to four different employees if every employee is assigned at least one job ? (8)

13. a) i) Check whether the following graphs are isomorphic or not. (6)



ii) If A is the adjacency matrix of a graph G with $V(G) = \{v_1, v_2, \ldots, v_p\}$, prove that for any $n \geq 1$, the $(i, j)^{th}$ entry of A^n is the number of $v_i - v_j$ walks of length n in G. (10)

(OR)

b) i) Define self complementary graph. Show that if G is a self complementary simple graph with n vertices then $n \equiv 0$ or $1 \pmod 4$. (6)

ii) Show that a simple graph G is Eulerian if and only if all its vertices have even degree. (10)

14. a) State and prove Lagrange's theorem on groups. (16)

(OR)

b) i) Show that a non empty subset H of a group (G, *) is a subgroup of G if and only if $a * b^{-1} \in H$ for all $a, b \in H$. (8)

ii) Show that the Kernel of a group homomorphism is a normal subgroup of the group. (8)

15. a) i) Show that every chain is a distributive lattice. (8)

ii) Let $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$ be the divisors of 100. Draw the Hasse diagram of $(D_{100}, /)$ where / is the relation "division".

Find (I) glb {10, 20} (II) lub {10, 20} (III) glb {5, 10, 20, 25}

(IV) lub {5, 10, 20, 25}. (8)

(OR)

b) i) In a Boolean Algebra, show that $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. (8)

ii) Define a modular lattice and prove that every distributive lattice is modular but not conversely. (8)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) In a distributive complemented lattice. Show that the following are equivalent.

(i) $a \leq b$ (ii) $a \wedge \bar{b} = 0$, (iii) $\bar{a} \vee b = 1$ (iv) $\bar{b} \leq \bar{a}$ (8)

OR

(b) (i) Show that the De Morgan's laws are valid in a Boolean Algebra. (8)

(ii) Show that every chain is modular. (8)

Reg. No. :

Question Paper Code : 57517

30/5/16 AN

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fifth Semester

Computer Science and Engineering

MA 6566 – DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Find the truth table for $p \rightarrow q$.

2. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.

3. How many different words are there in the word ENGINEERING ?

4. State the pigeon hole principle.

5. How many edges are there in a graph with 10 vertices each of degree 5 ?

6. Define self complementary graph.

7. Show that every cyclic group is abelian.

8. Let Z be the group of integers with the binary operation * defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.

9. Let $X = \{1, 2, 3, 4, 5, 6\}$ and R be a relation defined as $\langle x, y \rangle \in R$ if and only if $x - y$ is divisible by 3. Find the elements of the relation R .

10. Show that the absorption laws are valid in a Boolean algebra.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Show that '$\sqrt{2}$ is irrational'. **(6)**

(ii) Show that "It rained" is a conclusion obtained from the statements.

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded". **(10)**

OR

(b) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(7P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. **(8)**

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. **(8)**

12. (a) (i) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. **(8)**

(ii) Use generating function to solve the recurrence relation $S(n+1) - 2S(n) = 4^n$ with $S(0) = 1$, $n \geq 0$. **(8)**

OR

(b) (i) Using mathematical induction show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1} - 1}{2}$. **(8)**

(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex. **(8)**

13. (a) (i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. **(10)**

(ii) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) 1 (mod 4) vertices. **(6)**

OR

(b) (i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic ? **(10)**

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Prove that the number of odd degree vertices in any graph is even. **(6)**

14. (a) (i) In any group $\langle G, * \rangle$, show that $(a * b)^{-1} = b^{-1} * a^{-1}$, for all $a, b \in G$. **(6)**

(ii) State and prove Lagrange's theorem on groups. **(10)**

OR

(b) (i) Prove that every subgroup of a cyclic group is cyclic. **(8)**

(ii) Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. **(8)**

Reg. No. :

Question Paper Code : 72078

12/05/17 AN

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth Semester

Computer Science and Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the truth table for $p \rightarrow q$.

2. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.

3. How many different words are there in the word MATHEMATICS?

4. Find the minimum number of students need to guarantee that five of them belongs to the same subject, if there are five different major subjects.

5. How many edges are there in a graph with 10 vertices each of degree 3?

6. Give an example of self complementary graph.

7. Show that every cyclic group is abelian.

8. Let Z be the group of integers with the binary operation $*$ defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.

9. Let X = {1, 2, 3, 4, 5, 6} and R be a relation defined as $\langle x, y \rangle \in R$ if and only if $x - y$ is divisible by 3. Find the elements of the relation R.

10. Show that the absorption laws are valid in a Boolean algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. (8)

(ii) Use rules of inferences to obtain the conclusion of the following arguments :

"Babu is a student in this class, knows how to write programmes in JAVA". "Everyone who knows how to write programmes in JAVA can get a high-paying job". Therefore, "someone in this class can get a high-paying job". (8)

Or

(b) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (8)

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. (8)

12. (a) (i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2,3, 5 and 7. (8)

(ii) Solve the recurrence relation $a_n - 7a_{n-1} + 6a_{n-2} = 0$, for $n \geq 2$ with initial conditions $a_0 = 8$ and $a_1 = 6$, using generating function. (8)

Or

(b) (i) Using mathematical induction, show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1} - 1}{2}$. (8)

(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex. (8)

13. (a) (i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

(ii) Prove that the complement of a disconnected graph is connected. (6)

Or

(b) (i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic? (10)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Prove that the number of odd degree vertices in any graph is even. (6)

14. (a) State and prove Lagrange's theorem on groups. (16)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) In a distributive complemented lattice, show that the following are equivalent. (8)

(1) $a \leq b$

(2) $a \wedge \overline{b} = 0$

(3) $\overline{a} \vee b = 1$

(4) $\overline{b} \leq \overline{a}$.

Or

(b) Show that every ordered lattice $\langle L, \leq \rangle$ satisfies the following properties of the algebraic lattice (i) idempotent (ii) commutative (iii) Associative (iv) Absorption. (16)

Reg. No. :

Question Paper Code : 41320

14/05/18

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fifth Semester

Computer Science and Engineering

MA 6566 – DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Define proposition.
2. Give the symbolic form of "Some men are giant".
3. Define Pigeon hole principle.
4. How many permutations can be made out of letter or word 'COMPUTER' ?
5. Show that there does not exist a graph with 5 vertices with degrees 1, 3, 4, 2, 3 respectively.
6. Define Hamiltonian path.
7. Define semi group.
8. Prove that in a group idempotent law is true only for identity element.
9. Let A = {1, 2, 5, 10} with the relation divides. Draw the Hasse diagram.
10. Prove that a lattice with five elements is not a Boolean algebra.

PART – B (5×16=80 Marks)

11. a) i) Show that $(\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$, without using truth table. (8)

ii) Show that using Rule C.P, $\neg P \vee Q$, $\neg Q \vee R$, $R \rightarrow S \Rightarrow P \rightarrow S$ (8)

(OR)

b) i) Find the PCNF of $(P \vee R) \wedge (P \vee \neg Q)$ Also find its PDNF, without using truth table. (8)

ii) Show that $(\forall x)\,[P(x) \vee Q(x)] \Rightarrow (\forall x)\,P(x) \vee (\exists x)\,Q(x)$. (8)

12. a) i) Prove that $n^3 - n$ is divisible by 3 for $n \geq 1$ (8)

ii) Solve $G(k) - 7G(k-1) + 10G(k-2) = 8k + 6$. (8)

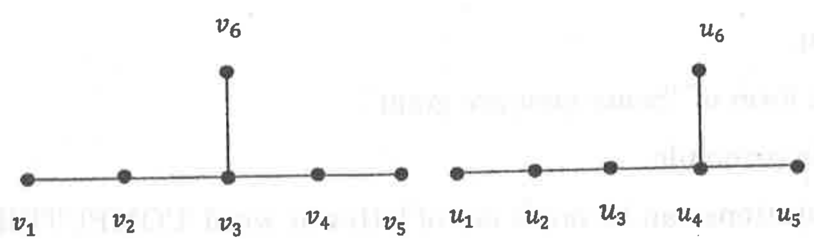
(OR)

b) i) Find the numbers between 1 to 250 that are not divisible by any of the integers 2 or 3 or 5 or 7. (8)

ii) Solve using generating functions : $S(n) + 3S(n-1) - 4S(n-2) = 0; n \geq 2$ given $S(0) = 3, S(1) = -2$. (8)

13. a) i) State and prove Hand shaking theorem. Hence prove that for any simple graph G with n vertices, the number of edges of G is less than or equal to $\frac{n(n-1)}{2}$. (8)

ii) Establish the isomorphism of the following pairs of graphs. (8)



(OR)

b) i) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2. (8)

ii) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. (8)

14. a) i) Show that $(Q^+, *)$ is an abelian group, where * is defined by $a * b = \frac{ab}{2}, \forall a, b \in Q^+$ (8)

ii) Prove that kernel of a homomorphism is a normal subgroup of G. (8)

(OR)

b) i) Prove that intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

ii) Let G be a finite group and H be a subgroup of G. Then prove that order of H divides order of G. (8)

15. a) i) Show that $(N, \leq)$ is a partially ordered set, where N is the set of all positive integers and $\leq$ is a relation defined by $m \leq n$ if and only if $n - m$ is a non-negative integer. (8)

ii) In a complemented and distributive lattice, prove that complement of each element is unique. (8)

(OR)

b) i) Let $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ with a relation $x \leq y$ if and only if x divides y. Find :

i) All lower bounds of 10 and 15

ii) GLB of 10 and 15

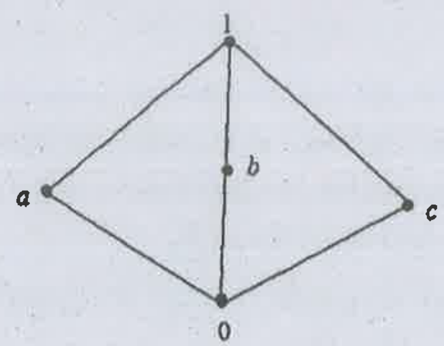
iii) All upper bound are 10 and 15

iv) LUB of 10 and 15

v) Draw the Hasse diagram for D_{30}. (8)

ii) Let $(L, , \vee, \wedge, \leq)$ be a distributive lattice and $a, b, \in L$ if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$. Then show that $b = c$. (8)

(b) (i) Examine whether the lattice given in the following Hasse diagram is distributive or not. (4)



(ii) If $P(S)$ is the power set of a non-empty S, prove that $\{P(S), Y, I, \setminus, \phi, S\}$ is a Boolean algebra. (12)

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18/05/15

Reg. No. :

Question Paper Code : 53255

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fifth Semester

Computer Science Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Give the contrapositive statement of the statement 'If there is rain, then I buy an umbrella".

2. Construct the truth table for $P \to \sim Q$.

3. Find the sequence whose generating function is $\frac{1}{1-9x^2}$.

4. How many ways the letters in the word "Committee" can be arranged?

5. How many edges are there in a graph with 10 vertices each of degree 3?

6. Give an example of self complementary graph.

7. Prove that identity element in a group is unique.

8. Prove that every cyclic group is abelian.

9. Let $X = \{1, 2,3,4,5, 6\}$ and R be a relation defined as $x, y \in R$ if and only if $x - y$ is divisible by 3. Find the elements of the relation R.

10. Show that the absorption laws are valid in a Boolean algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow R)$ by using equivalences. (8)

(ii) Use rules of inferences to obtain the conclusion of the following arguments :

"Babu is a student in this class, knows how to write programmes in JAVA'. 'Everyone who knows how to write programmes in JAVA can get a high-paying job'. Therefore, 'someone in this class can get a high-paying job". (8)

Or

(b) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (8)

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q. (8)

12. (a) (i) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. (8)

(ii) Use generating function to solve the recurrence relation $S(n+1) - 2S(n) = 4^n$ with $S(0) = 1, n \geq 0$. (8)

Or

(b) (i) Using mathematical induction show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1}-1}{2}$. (8)

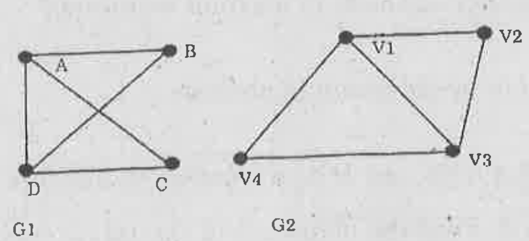
(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if

(1) they can be male or female,

(2) two must be men and two women,

(3) they must all are of the same sex. (8)

13. (a) (i) Establish the isomorphism for the following graphs. (8)



(ii) Prove that a graph G is disconnected if and only if the vertex set V is partitioned into two non-empty subsets U and W such that there exists no edge in G whose one vertex is in U and one vertex is in W. (8)

Or

(b) (i) Show that K_n has a Hamiltonian cycle for $n > 3$. What is the maximum number of edge disjoint cycles possible in K_n? Obtain all the edge disjoint cycles in K_7. (8)

(ii) Prove that maximum number of edges in a bipartite graph with n vertices is $\frac{n^2}{4}$. (8)

14. (a) (i) Show that $(Q^+, *)$ is an abelian group, where $*$ is defined by $a * b = \frac{ab}{2}, \forall a, b \in Q^+$. (8)

(ii) Let $f : (G, *) \rightarrow (G', \Delta)$ be a group homomorphism Then prove that

(1) $[f(a)]^{-1} = f(a^{-1}) \forall a \in G$.

(2) $f(e)$ is an identity of G', when e is an identity of G. (8)

Or

(b) (i) Prove that the intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

(ii) State and prove Lagrange's theorem in a group. (8)

15. (a) (i) Let $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and let the relation R be divisor on D_{30}.

Find

(1) all the lower bounds of 10 and 15

(2) the glb of 10 and 15

(3) all upper bound of 10 and 15

(4) the lub of 10 and 15

(5) draw the Hasse diagram. (8)

(ii) Prove that in a Boolean algebra $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (8)

Or

PART – B (5×16=80 Marks)

11. a) i) Show that the following two statements are logically equivalent : "It is not true that all comedians are funny" and "There are some comedians who are not funny". (8)

ii) Prove that the conditional statement $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is a tautology using logical equivalences. (8)

(OR)

b) i) Use rules of inference to prove that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passe the first exam has not read the book". (8)

ii) In an island there are two kind of inhabitants Knights (who always tell the truth) and their opposites, Knaves (who always lie). Let A and B be any two people from that island. A says "B is a knight" and B says "The two of us are opposite types". Define exhaustive proof strategy and use it to find the nature of A and B. (8)

12. a) A valid code word is an n-digit decimal number containing even number of 0's. If a_n denotes the number of valid code words of length n then find an explicit formula for a_n using generating functions. (16)

(OR)

b) i) If H_n denote harmonic numbers, then prove that $H_{2^n} \geq 1 + \frac{n}{2}$ using mathematical induction. (10)

ii) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French and Russian, how many students have taken a course in all three languages ? (6)

13. a) i) Examine whether the following two graphs G and G′ associated with the following adjacency matrices are isomorphic.

$$\begin{pmatrix} 0&1&0&1&0&0\\ 1&0&1&0&0&1\\ 0&1&0&1&0&0\\ 1&0&1&0&1&0\\ 0&0&0&1&0&1\\ 0&1&0&0&1&0 \end{pmatrix} \text{ and } \begin{pmatrix} 0&1&0&0&1&0\\ 1&0&1&0&0&0\\ 0&1&0&1&0&1\\ 0&0&1&0&1&0\\ 1&0&0&1&0&1\\ 0&0&1&0&1&0 \end{pmatrix}$$ (10)

ii) Discuss the various graph invariants preserved by isomorphic graphs. (6)

(OR)

b) i) Prove that a simple graph with n vertices and k components can not have more than $\frac{(n-k)(n-k+1)}{2}$ edges. (10)

ii) Prove that a simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color. (6)

14. a) If G is a group of order n and H is a sub-group of G of order m, then prove the following results :

i) $a \in G$ is any element, then the left coset aH of H in G consists of as many elements as in H. (4)

ii) Any two left cosets of H in G is either equal or disjoint. (8)

iii) The index of H in G is an integer. (4)

(OR)

b) i) Examine whether $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0 \in R \right\}$ is a commutative group under matrix multiplication, where R is the set of all real numbers. (10)

ii) Prove that (Z_5, X_5) is a commutative monoid, where X_5 is the multiplication modulo 5. (6)

15. a) i) Let $\langle L, \leq \rangle$ be a lattice in which * and $\oplus$ denote the operations of meet and join respectively. For any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$. (8)

ii) Prove that every chain is a distributive lattice. (8)

(OR)

b) i) In a Boolean algebra B, if $a, b, c \in B$, then prove that $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. (12)

ii) Let $\langle L, *, \oplus \rangle$ and $\langle S, \wedge, \vee \rangle$ be any two lattices with the partial orderings $\leq$ and $\leq'$ respectively. If g is a lattice homomorphism, then g preserves the partial ordering. (4)

16.11.17 FN

Reg. No. :

Question Paper Code : 50789

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fifth Semester

Computer Science and Engineering

MA 6566 – DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three Hours　　　　Maximum : 100 Marks

Answer ALL questions

PART – A　　　　(10×2=20 Marks)

1. Find the truth value of $\forall x(x^2 \geq x)$ if the universe of discourse consists of all real numbers. Also write the negation of the given statement.

2. If the universe of discourse consists of all real numbers then translate the following formula into a logical statement :

$\forall x \exists y \forall z\left[(x > 0) \to [y^2 = x = (-y)^2]\right] \wedge \left[(z^2 = x) \to (z = y \vee -y)\right]$

3. How many cards must be selected from a standard deck of 52 cards (4 different suits of equal size) to guarantee that at least three cards of the same suit are chosen ?

4. Write the particular solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + 3^n$.

5. Draw a graph that is an Euler graph but not Hamiltonian.

6. Can you draw a graph of 5 vertices with degree sequence 1, 2, 3, 4, 5 ?

7. Define 'kernel of homomorphism' in a group.

8. If $\langle R, +, . \rangle$ is a ring then prove that $a . 0 = 0, \forall a \in R$ and 0 is the identity element in R under addition.

9. State modular inequality of lattices.

10. Write the only Boolean algebra whose Hasse diagram is a chain.

Reg. No. :

Question Paper Code : 20758

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fifth Semester

Computer Science and Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Write the contra positive of the implication. "If it is Sunday then it is holiday".
2. Show that the propositions $p \to q$ and $\neg p \vee q$ are equivalent.
3. How many cards must be selected from a deck of 52 cards to guarantee that at least three cards of the same suit are chosen?
4. How many bit strings of length 12 contain exactly four 1s?
5. Show that the number of odd degree vertices in a simple graph is even.
6. Give an example of a graph which is both Eulerian and Hamiltonian.
7. Define a semigroup and give an example.
8. Show that in a group $(G,*)$ if for any $a,b \in G$, $(a*b)^2 = a^2 * b^2$, then $(G,*)$ is abelian.
9. Draw the Hasse diagram of $(S_{24},/)$ where S_{24} denotes the set of positive divisors of 24 and / denotes the relation "division".
10. Prove that in a lattice $(L,\leq)$, $a*(a \oplus b) = a$ where $*$ and $\oplus$ denote the meet and join.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Translate the statement $\forall x\,(C(x) \vee \exists y\,(C(y) \wedge F(x,y)))$ into English, where $C(x)$ is "x has a computer", $F(x,y)$ is "x and y are friends" and the universe of discourse for both x and y consists of all students in your class. (4)

(ii) Translate the statement "The sum of two positive integers is a positive integer" into a logical expression. (4)

(iii) Show that the premises, "A student in this class has not read the book" and "Everyone in this class passed the exam" imply the conclusion "Someone who passed the exam has not read the book". (8)

Or

(b) (i) Obtain the principal disjunctive and conjunctive normal forms of the formula $(\sim p \to r) \wedge (q \leftrightarrow p)$. (8)

(ii) Using proof by contradiction, prove that $\sqrt{2}$ is irrational. (8)

12. (a) (i) Use mathematical induction to show that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer. (8)

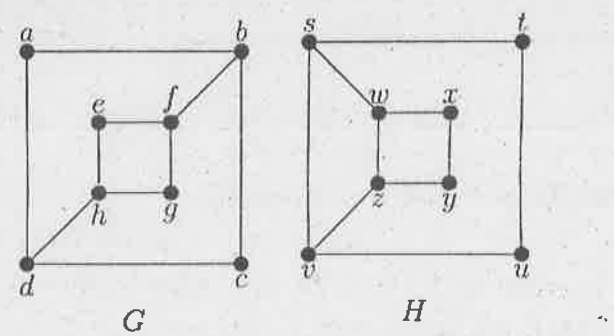
(ii) Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}$ with $f_0 = 0$; $f_1 = 1$. (8)

Or

(b) (i) Using generating functions, solve $a_n = 8a_{n-1} + 10^{n-1}$ with $a_0 = 1$; $a_1 = 9$. (8)

(ii) How many onto functions are there from a set with six elements to set with three elements? (8)

13. (a) (i) Determine whether the graphs given below are isomorphic. (8)



(ii) Let G be a simple graph with adjacency matrix A. Show that the number of different walks of length r from v_i to v_j, where r is a positive integer, equals the $(i,j)^{th}$ entry of A^r. (8)

Or

(b) (i) Show that a connected simple graph is Eulerian if and only if all its vertices have even degree. (8)

(ii) Represent each of the following graphs with an adjacency matrix.

(1) K_4

(2) $K_{1,4}$

(3) C_4

(4) W_4. (8)

14. (a) (i) State and prove Lagrange's theorem on groups. (12)

(ii) Show that if every element in a group is its own inverse, then the group must be abelian. (4)

Or

(b) (i) Show that a subset $S \neq \phi$ of G is a subgroup of the group $(G,*)$ if and only if for any pair of elements $a, b \in S$, $a * b^{-1} \in S$. (8)

(ii) Let f be a group homomorphism from $(G,*)$ to (H,Δ). Define Kernel of f and show that it is a subgroup of $(G,*)$. (8)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) Show that every distributive lattice is modular, but not conversely. (8)

Or

(b) (i) Show that the following are equivalent in a Boolean Algebra

$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$. (8)

(ii) In a Boolean algebra, prove that $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. (8)

Reg. No. :

Question Paper Code : 91790

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019
Fifth Semester
Computer Science and Engineering
MA 6566 – DISCRETE MATHEMATICS
(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Find the truth table for $p \rightarrow q$.

2. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.

3. State the pigeonhole principle.

4. Find the number of permutations of the letters in the word M I S S I S S I P P I ?

5. Draw the complete bipartite graph $K_{3,4}$.

6. State hand shaking theorem.

7. Show that every cyclic group is abelian.

8. Let Z be the group of integers with the binary operation $*$ defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.

9. Define a lattice.

10. State the De Morgan's laws of Boolean Algebra.

PART – B (5×16=80 Marks)

11. a) i) Prove that the premises $P \to Q$, $Q \to R$, $R \to S$, $S \to \sim R$ and $P \wedge S$ are inconsistent. (8)

ii) Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "Someone in this class can get a high-paying job". (8)

(OR)

b) i) Without constructing the truth tables, obtain the principle disjunctive normal form of $(\sim P \to R) \wedge (Q \leftrightarrow P)$. (8)

ii) Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$ and Q. (8)

12. a) i) Prove that $\frac{1}{1.2}+\frac{1}{2.3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$ using principle of induction. (8)

ii) How many integers between 1 to 300 are there that are divisible by,

1) at least one of 3, 5, 7

2) 3 and 5 but not by 7

3) 5 but not 3 and 7. (8)

(OR)

b) i) A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if

1) They can be of any colour

2) Two must be white and two red

3) They must all be of the same color. (8)

ii) Solve $D(k) - 7D(k-2) + 6D(k-3)=0$, where $D(0) = 8$, $D(1) = 6$ and $D(2) = 22$. (8)

13. a) i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

ii) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) 1(mod 4) vertices. (6)

(OR)

b) i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic ? (10)

$$\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&1&0\\0&1&0&1&0&1\\0&0&1&0&1&0\\0&1&0&1&0&1\\1&0&1&0&1&0\end{bmatrix}\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&0&1\\0&1&0&1&1&0\\0&0&1&0&1&0\\0&0&1&1&0&1\\1&1&0&0&1&0\end{bmatrix}$$

ii) Prove that the number of odd degree vertices in any graph is even. (6)

14. a) State and prove Lagrange's theorem on groups. (16)

(OR)

b) i) Prove that every subgroup of a cyclic group is cyclic. (8)

ii) Let $f: G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

15. a) i) In a complemented and distributive lattice, prove that complement of each element is unique. (8)

ii) Prove that every chain is a distributive lattice. (8)

(OR)

b) i) Consider the Lattice D_{105} with the partial ordered relation, "divides" then

1) Draw the Hasse diagram of D_{105}.

2) Find the complement of each elements of D_{105}.

3) Find the set of atoms of D_{105}.

4) Find the number of subalgebras of D_{105}. (8)

ii) Show that in a Boolean algebra $a \leq b \Leftrightarrow a \wedge \overline{b} = 0 \Leftrightarrow \overline{a} \vee b = 1 \Leftrightarrow \overline{b} \leq \overline{a}$. (8)