SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

DEPARTMENT OF AEROSPACE ENGINEERING

Subject Code & Name**: 19AST302 FLIGHT DYNAMICS** **Date: 26.11.2023**

DAY:**39** TOPIC: **ROUTH'S DISCRIMINANT- Modes of stability**

The solutions to the simultaneous differential equations (10–66) can be obtained by solving them as they stand. However, it has been found convenient and mathematically simpler to solve these equations in two steps. The first step is to consider the elevator fixed, which eliminates the $\Delta\delta_e$ terms in (10–66*c*) and all of equation (10–66*d*), while the second step is to consider the elevator free, with the speed assumed constant, thereby eliminating all the terms containing u as a variable and all of equation (10–66*a*). This breakdown is allowable because it has been shown through experience that the important modes of airplane motion can be obtained in this manner with only small losses in accuracy.

The elevator-fixed condition will be assumed first, and the nature of the airplane's motion after a disturbance with controls locked will be investigated by solution of the following reduced equations of motion:

$$(C_D + \mathrm{d})u + \frac{1}{2}(C_{D_\alpha} - C_L)\alpha + \frac{C_L}{2}\theta = 0$$

$$C_L u + (\tfrac{1}{2}C_{L_\alpha} + \mathrm{d})\alpha - \mathrm{d}\theta = 0$$

$$(C_{m_\alpha} + C_{m_{\mathrm{d}\alpha}}\mathrm{d})\alpha + (C_{m_{\mathrm{d}\theta}}\mathrm{d} - h\mathrm{d}^2)\theta = 0$$

The solutions to these equations of motion are obtained by assuming the solution in the form:

$$u = u_1 e^{\lambda t/\tau} \quad \alpha = \alpha_1 e^{\lambda t/\tau} \quad \theta = \theta_1 e^{\lambda t/\tau}$$

where λ is a real or complex constant of equal value for each variation, and where u_1, α_1, and θ_1 are also real or complex constants.

If the above values for the variables are substituted into equation (10–89), together with their first and second derivatives (i.e., $\mathrm{d}u = u_1\lambda e^{\lambda t/\tau}$ and $d^2u = u_1\lambda^2 e^{\lambda t/\tau}$, etc.) and the common term $e^{\lambda t/\tau}$ divided out, equation (10–89) reduces to three algebraic equations in the unknown λ, and the new variables u_1, α_1, and θ_1.

$$(C_D + \lambda)u_1 + \frac{1}{2}(C_{D\alpha} - C_L)\alpha_1 + \frac{C_L}{2}\theta_1 = 0$$

$$C_L u_1 + (\tfrac{1}{2}C_{L\alpha} + \lambda)\alpha_1 - \lambda\theta_1 = 0$$

$$(C_{m\alpha} + C_{md\alpha}\lambda)\alpha_1 + (C_{md\theta}\lambda - h\lambda^2)\theta_1 = 0$$

These equations are now homogeneous algebraic equations, and as such are subject to the requirement that, for them to be consistent, the determinant of their coefficients must vanish.

$$\begin{vmatrix} C_D + \lambda & \tfrac{1}{2}(C_{D\alpha} - C_L) & \frac{C_L}{2} \\ C_L & (\tfrac{1}{2}C_{L\alpha} + \lambda) & -\lambda \\ 0 & (C_{m\alpha} + C_{md\alpha}\lambda) & (C_{md\theta}\lambda - h\lambda^2) \end{vmatrix} = 0$$

By expanding this determinant a quartic equation in λ is obtained, the roots of which are the four values of λ that determine the final solution.

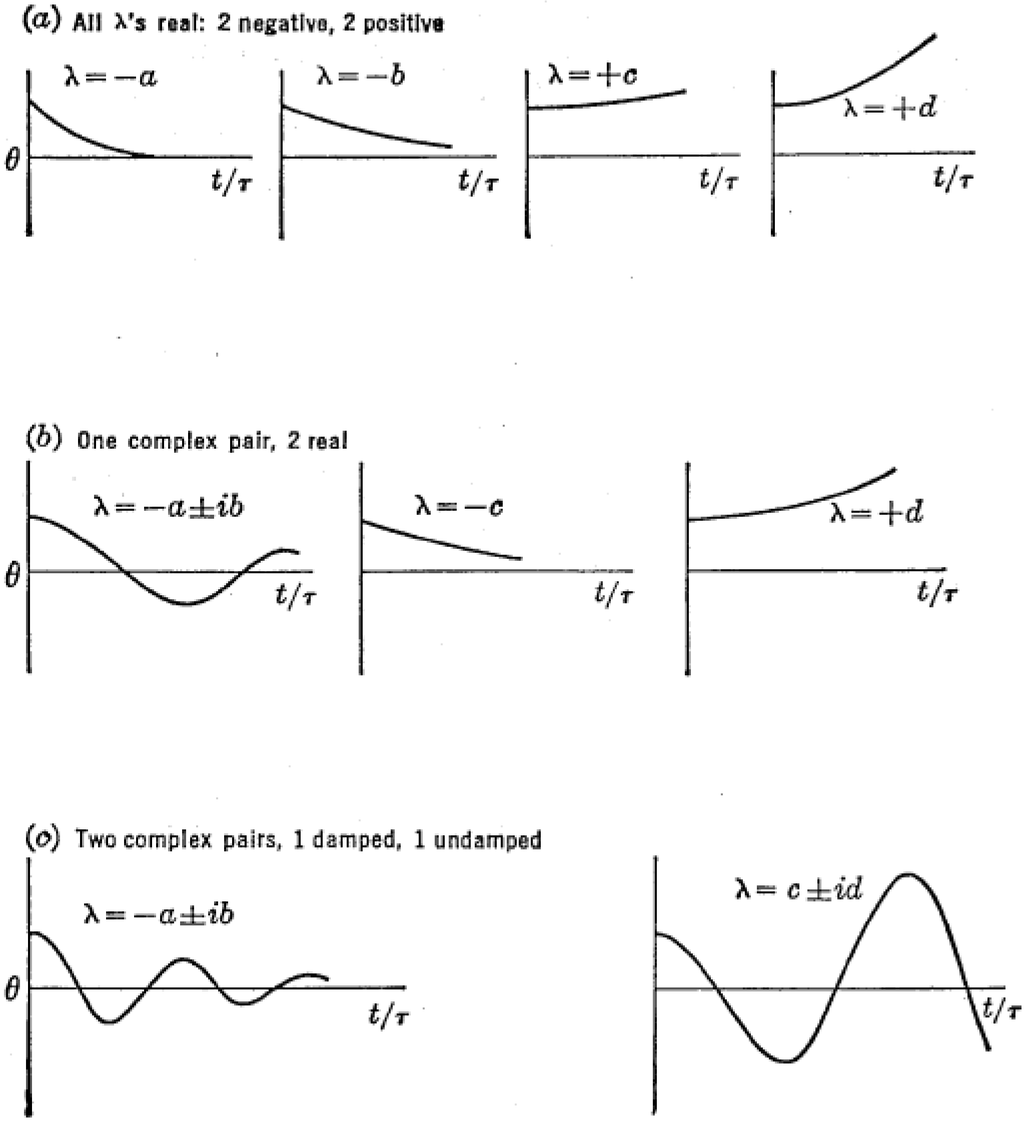
$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E$$

$$u = u_1 e^{\lambda_1 t/\tau} + u_2 e^{\lambda_2 t/\tau} + u_3 e^{\lambda_3 t/\tau} + u_4 e^{\lambda_4 t/\tau} \text{ etc.}$$

In order to study the characteristic motions of any dynamic system, it is not necessary to establish the values of the coefficients u_1, u_2, u_3, and u_4, as these will vary only as a function of the initial disturbance when $t/\tau = 0$. The thing that is important is to determine the character of the motion, if oscillatory the period and damping, and if aperiodic the rate of convergence or divergence. This can be accomplished readily by investigating the four values of λ obtained from the quartic (10–93).

If all the λ's come out as real numbers, the motion is aperiodic, convergent if negative, divergent if positive. If any of the λ's form a complex pair, the motion is oscillatory, damped if the real part is negative, undamped if the real part is positive. This is demonstrated in Figure 10–11.

The study, then, of the character of the motion of the airplane after a disturbance from equilibrium is a study of the roots of the quartic



equation in λ (10–93). This equation is often referred to as the stability quartic.

The coefficients of the stability quartic can be obtained by expansion of the determinant in λ (10–93) and collection of like powers of λ. The result of this expansion yields the following:

$$A = 1$$

$$B = \frac{1}{2} C_{L\alpha} + C_D - \frac{1}{h} C_{m\mathrm{d}\theta} - \frac{1}{h} C_{m\mathrm{d}\alpha}$$

$$C = \frac{1}{2} C_D C_{L\alpha} + \frac{C_L{}^2}{2} - \frac{C_{m\mathrm{d}\theta}}{2h} C_{L\alpha} - \frac{C_D}{h} C_{m\mathrm{d}\theta} - \frac{C_L}{2} C_{D\alpha} - \frac{1}{h} C_{m\alpha} - \frac{C_{m\mathrm{d}\alpha}}{h} C_D$$

$$D = \frac{C_L}{2h} C_{D\alpha} C_{m\mathrm{d}\theta} - \frac{C_D}{2h} C_{m\mathrm{d}\theta} C_{L\alpha} - \frac{C_L{}^2}{2h} C_{m\mathrm{d}\alpha} - \frac{C_L{}^2}{2h} C_{m\mathrm{d}\theta} - \frac{C_D}{h} C_{m\alpha}$$

$$E = -\frac{C_L{}^2}{2h} C_{m\alpha}$$

Once the coefficients A–E above, of the quartic in λ, are determined for any given airplane and flight conditions, inspection of the quartic will yield valuable information concerning the motion before the roots are obtained. If all the coefficients are positive, there can be no positive real root and there is no possibility of a pure divergence. If a combination of the coefficients known as Routh's discriminant $(BCD - AD^2 - B^2E)$, is positive, then there is no possibility of the real part of any complex pair being positive and there will be no undamped oscillation. If Routh's discriminant is equal to zero, there will be a neutrally damped oscillation; if negative, there will be one complex pair with a positive real part, implying an undamped oscillation. If the term $E = 0$, then, one of the roots is zero and one of the modes can continue unchanged indefinitely; while if one of the coefficients is negative, there can be either an increasing oscillation or a pure divergence in one of the modes.