ANALYSIS OF DISCRETE TIME SIGNAL ANALYSIS

Discrete time Fourier Transform :- [Fourier Transform of Discrete sequence]

Discrete time signals are analysed with the help of periodic signals.

DTFT → Both periodic and Non-periodic signals.

Definition :-

DTFT of the discrete time signal $x(n)$ is given as

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \rightarrow \text{Analysis equation}$$

Here ω is the frequency of discrete time signal. The range of ω from $-\pi$ to π are equivalently $(0, 2\pi)$ and ω is continuous over this range. $X(\omega)$ is also called as spectrum of discrete time signal.

Inverse DTFT :-

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega \rightarrow \text{synthesis equation}$$

Existence of DTFT :-

DTFT of $x(n)$ will converge if $x(n)$ is absolutely summable

$$\sum_{n=-\infty}^{\infty} |x(n)| < \infty$$

Properties of DTFT :-

(i) periodicity :-

If $x(n) \leftrightarrow X(\omega)$ then $X(\omega + 2\pi k) = X(\omega)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$X(\omega+2\pi k) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\omega+2\pi k)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}\, e^{-j2\pi kn}$$

$$= X(\omega) \cdot e^{-j2\pi kn}$$

$$X(\omega+2\pi k) = X(\omega)$$

2) Linearity :-

$x(n) \leftrightarrow X(\omega)$, $y(n) \leftrightarrow Y(\omega)$ then

$$a\, x_1(n) + b\, y(n) = a\, X(\omega) + b\, Y(\omega)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} [a\, x(n) + b\, y(n)]\, e^{-j\omega n}$$

$$= a \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} + b \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\omega n}$$

$$= a\, X(\omega) + b\, Y(\omega)$$

3) Time shifting :-

$x(n) \leftrightarrow X(\omega)$ then $x(n-n_0) \leftrightarrow e^{-j\omega n_0} X(\omega)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n-n_0)\, e^{-j\omega n}$$

$n - n_0 = m$ $\qquad$ $n = m + n_0$

$$= \sum_{n=-\infty}^{\infty} x(m)\, e^{-j\omega(m+n_0)}$$

$$= \sum_{n=-\infty}^{\infty} x(m)\, e^{-j\omega m}\, e^{-j\omega n_0}$$

$$= X(\omega)\, e^{-j\omega n_0}$$

4) Frequency shifting :-

$$x(n) \leftrightarrow X(\omega) \quad \text{then} \quad e^{j\omega_0 n}\, x(n) \leftrightarrow X(\omega-\omega_0)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{j\omega_0 n}\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\omega-\omega_0)n}$$

$$= X(\omega-\omega_0)$$

5) Scaling property :-

$$x(n) \leftrightarrow X(\omega) \quad \text{then} \quad x(np) \leftrightarrow X(\omega/p)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(np)\, e^{-j\omega n}$$

$$np = m \;, \quad n = m/p$$

$$= \sum_{n=-\infty}^{\infty} x(m)\, e^{-j\omega(m/p)}$$

$$= \sum_{n=-\infty}^{\infty} x(m)\, e^{-j(\omega/p)\, m}$$

$$X(\omega) = X(\omega/p)$$

6) Differentiation in time Domain :-

$$x(n) \leftrightarrow X(\omega) \quad \text{then} \quad -jn\, x(n) \leftrightarrow \frac{d}{d\omega} X(\omega)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$\frac{d}{d\omega} X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, -jn\; e^{-j\omega n}$$

$$\frac{d}{d\omega} X(\omega) = -jn\, x(n)$$

7) convolution in time Domain :-

$$x(n) \leftrightarrow X(\omega), \quad y(n) \leftrightarrow Y(\omega) \quad \text{then}$$

$$x(n) * y(n) \leftrightarrow X(\omega)\; Y(\omega)$$

$$\boxed{x(n) * y(n) = \sum_{n=-\infty}^{\infty} x(m)\, y(n-m)}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[x(n) * y(n)\right] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} x(m)\, y(n-m)\right] e^{-j\omega n}$$

$$n - m = \tau, \quad n = \tau + m$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} x(m)\, y(\tau)\right] e^{-j\omega(\tau+m)}$$

$$= \sum_{n=-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} x(m)\, y(\tau)\right] e^{-j\omega\tau}\, e^{-j\omega m}$$

$$= \sum_{n=-\infty}^{\infty} x(m)\, e^{-j\omega m} \sum_{n=-\infty}^{\infty} y(\tau)\, e^{-j\omega\tau}$$

$$= X(\omega)\; Y(\omega)$$

8) Multiplication in Time D ... i :-

$x(n) \leftrightarrow X(\omega)$ and $y(n$ ... $Y(\omega)$ then

$$x(n) \cdot y(n) \leftrightarrow \frac{1}{2\pi} \quad (\omega) * Y(\omega)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(r \quad \omega n$$

$$= \sum_{n=-\infty}^{\infty} x(n) \cdot y(n)\, e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2\pi} \int_{-\pi}^{\pi} x(m) \, e^{j\omega n} \, dm \; y(n) \, e^{-j\omega n}$$

$$= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} x(m) \int_{-\pi}^{\pi} y(n) \, e^{-j(\omega-m)n} \, dm$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} x(m) \sum_{n=-\infty}^{\infty} y(n) \, e^{-j(\omega-m)n} \, dm$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} x(m) \, Y(\omega-m) \, dm$$

$$= \frac{1}{2\pi} X(\omega) * Y(\omega)$$

Parseval's Theorem :-

$x(n) \leftrightarrow X(\omega)$ then energy of signal is given as

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} x(n) \, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} X^*(\omega) \, e^{j\omega n} \, d\omega \right]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) \, X^*(\omega) \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega \, .$$