Power Series Expansion (or) Long Division Method :-

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \{ \dots + x(-2) z^{2} + x(-1) z^{1} + x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + \dots \}$$

$$x(n) = \{ \dots, x(-2), x(-1), x(0), x(1), x(2) \}$$

$x(n)$ can be obtained by collecting the co-efficients of z in $X(z)$ expansion

NOTE :-

1) when the ROC is $|z| > |a|$ (causal s/m) then expand $X(z)$ such that the powers of z are negative

2) when the ROC is $|z| < |a|$ (Non-causal s/m) then expand $X(z)$ such that the powers of z are positive.

① Find the inverse z-transform using power series expansion method :-

$$X(z) = \frac{1}{1-az^{-1}} \qquad ROC : |z| > |a|$$

$$
\begin{array}{r|l}
 & 1 + az^{-1} + a^2 z^{-2} \\
\hline
1 - az^{-1} & \cancel{1} \\
 & \cancel{1} - az^{-1} \\
 & (-) \quad (+) \\
\hline
 & \cancel{az^{-1}} \\
 & + \cancel{az^{-1}} - a^2 z^{-2} \\
 & (-) \quad (+) \\
\hline
 & \cancel{a^2 z^{-2}} \\
 & \cancel{a^2 z^{-2}} - a^3 z^{-3} \\
 & (-) \quad (+) \\
\hline
 & a^3 z^{-3}
\end{array}
$$

$$X(z) = \{ 1 + a z^{-1} + a^2 z^{-2} + \dots \}$$

$$x(n) = \{ 1 + a + a^2 + a^3 + \dots \}$$

$\therefore$ causal system

(ii) $$X(z) = \frac{1}{1-az^{-1}} \quad ; \quad \text{ROC}: |z| < |a|$$

$$-a^{-1}z - a^{-2}z^2 - a^{-3}z^3$$

$-az^{-1}+1$)

$\cancel{1}$

$\cancel{1} - a^{-1}z$

(−) (+)

$a^{-1}\cancel{z}$

$a^{-1}\cancel{z} \quad - a^{-2}z^2$

(−) (+)

$\cancel{a^{-2}z^2}$

(−) $\cancel{a^{-2}z^2} \quad - a^{-3}z^3$

(+)

$a^{-3}z^3$

$$X(z) = \{ \dots\dots -a^{-3}z^3 - a^{-2}z^2 - a^{-1}z \}$$

$$x(n) = \{ \dots\dots -a^{-3}, -a^{-2}, -a^{-1} \}$$

$$x(n) = -a^n u(-n-1)$$

Non-causal system

HW

2) $$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$ when $x(n)$ is causal & Non-causal

$x(n) = \{1, 4, 7, 10 \dots\}$ → causal

$x(n) = \{+ \dots 11, 8, 5, 2, \underset{\uparrow}{0}\}$ → Non-causal

3) $X(z) = \frac{z+1}{z^2-3z+2}$ when $x(n)$ is causal & Non-causal

(i)

$$\begin{array}{r|l} & z^{-1} + 4z^{-2} + 10z^{-3} + 22z^{-4} + \cdots \\ \hline z^2-3z+2 & \cancel{z} + 1 \\ & \cancel{z} - 3 + 2z^{-1} \\ & (-)\ (+)\ (-) \\ \hline & \cancel{4} - 2z^{-1} \\ & \cancel{4} - 12z^{-1} + 8z^{-2} \\ & (-)\ (+)\ (-) \\ \hline & \cancel{10z^{-1}} - 8z^{-2} \\ & (-)\cancel{10z^{-1}} - 30z^{-2} + 20z^{-3} \\ & (+)\ (-) \\ \hline & 22z^{-2} - 20z^{-3} \end{array}$$

$X(z) = \{ z^{-1} + 4z^{-2} + 10z^{-3} + 22z^{-4} + \cdots \}$

$x(n) = \{ 0, 1, 4, 10, 22, \cdots \}$

(ii) Non-causal :-

$$\begin{array}{r|l} & \frac{1}{2} + \frac{5}{4}z + \frac{13}{8}z^2 \\ \hline 2-3z+z^2 & \cancel{1} + z \\ & \cancel{1} - \frac{3}{2}z + \frac{1}{2}z^2 \\ & (-)\ (+)\ (-) \\ \hline & \cancel{\frac{5}{2}z} - \frac{1}{2}z^2 \\ & \cancel{\frac{5}{2}z} - \frac{15}{4}z^2 + \frac{5}{4}z^3 \\ & (-)\ (+)\ (-) \\ \hline & \cancel{\frac{13}{4}z^2} - \frac{5}{4}z^3 \\ & \cancel{\frac{13}{4}z^2} - \frac{39}{8}z^3 + \frac{13}{8}z^4 \\ & (-)\ (+)\ (-) \\ \hline & \frac{29}{8}z^3 - \frac{13}{8}z^4 \end{array}$$

$X(z) = \{ + \cdots \frac{13}{8}z^2 + \frac{5}{4}z + \frac{1}{2} \}$

$x(n) = \{ + \cdots \frac{13}{8}, \frac{5}{4}, \frac{1}{2}, 0 \}$