Inverse Z-Transform :-

1) partial fraction Method
2) Power series expansion Method (or) Long Division Method

Steps :-

1) convert $X(z)$ to the posible powers of z
2) Bring $X(z)$ to $X(z)/z$
3) Apply partial fraction Method
4) Multiply with z
5) Take Inverse Z-transform based on ROC

1) Find Inverse z-transform of $X(z) = \frac{z+4}{z^2-4z+3}$

$$X(z) = \frac{z(z+4)}{z(z^2-4z+3)}$$

$$\frac{X(z)}{z} = \frac{z+4}{z(z-1)(z-3)}$$

$$\frac{z+4}{z(z-1)(z-3)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-3}$$

$$z+4 = A(z-1)(z-3) + B(z)(z-3) + C(z)(z-1)$$

put $z=1$

$5 = B(-2)$

$\boxed{B = -5/2}$

put $z=0$

$4 = A(-1)-3$

$\boxed{A = 4/3}$

put $z=3$

$7 = C(3)(2)$

$\boxed{C = 7/6}$

$$\frac{X(z)}{z} = \frac{4}{3z} - \frac{5}{2(z-1)} + \frac{7}{6(z-3)}$$

$$X(z) = \frac{4z}{3z} - \frac{5z}{2(z-1)} + \frac{7z}{6(z-3)}$$

$$x(n) = \frac{4}{3}\delta(n) - \frac{5}{2}u(n) + \frac{7}{6}(3)^n u(n)$$

Find the Inverse z-transform of $X(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$

ROC : $|z| > 1$

$$X(z) = \frac{1}{(1+1/z)(1-1/z)^2}$$

Multiply & Divide by z^2

$$X(z) = \frac{z^2}{z^2} \frac{1}{(1+1/z)(1-1/z)^2}$$

$$X(z) = \frac{1}{\left(\frac{z+1}{z}\right) \frac{(z-1)^2}{\cancel{z^2}}} \cdot \frac{z^2}{\cancel{z^2}}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z+1)(z-1)^2}$$

$$\frac{z^2}{(z+1)(z-1)^2} = \frac{A}{z+1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$z^2 = A(z-1)^2 + B(z+1)(z-1) + C(z+1)$$

put $z=1$

$1 = C(2)$

$\boxed{C = 1/2}$

put $z=-1$

$1 = A(4)$

$\boxed{A = 1/4}$

put $z=0$

$0 = 1/4 + B(-1) + 1/2$

$\boxed{B = 3/4}$

$$\frac{X(z)}{z} = \frac{1}{4(z+1)} + \frac{3}{4(z-1)} + \frac{1}{2(z-1)^2}$$

$$X(z) = \frac{z}{4(z+1)} + \frac{3z}{4(z-1)} + \frac{z}{2(z-1)^2}$$

$$x(n) = \frac{1}{4} u(n) + \frac{3}{4} u(n) + \frac{1}{2} n\, u(n)$$