



CONVOLUTION SUMS : [Linear Convolution]

convolution of sum is defined as

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

Four steps involved in computing convolution :-

- * Folding * Shifting * Multiplication * Summation

Four methods available in compute convolution sum :-

- 1) Definition Method 2) Graphical Method
- 3) Tabulation Method 4) Multiplication Method

Let M be the total no of samples of x(n) and N be the total no of samples of h(n) then the total no of samples in y(n) = M+N-1

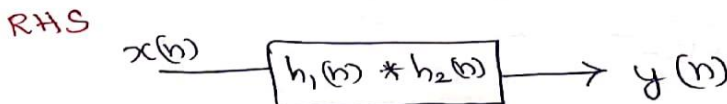
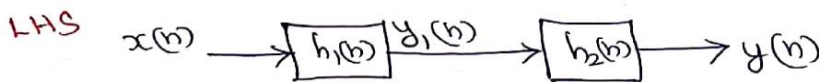
Properties :-

1) commutative property :-

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

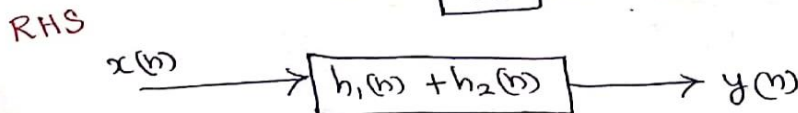
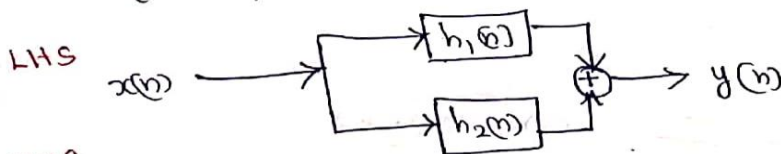
2) Associative property :-

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$



3) Distributive property :-

$$x(n) * h_1(n) + x(n) * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$





1) Find the convolution sum of the given sequence :-

$$x(n) = \{1, 2, 3, 4, 5\} \quad h(n) = \{6, 7, 8\}$$

Multiplication Method :-

	1	2	3	4	5		
			6	7	8		
<hr/>							
		8	16	24	32	40	
●	7	14	21	28	35		
	6	12	18	24	30		
<hr/>							
	6	19	40	61	82	67	40
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$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$

Tabulation method :-

		6	7	8	
1		6	7	8	
2		12	14	16	
3		18	21	24	
4		24	28	32	
5		30	35	40	

$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$



Definition Method :-

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$x(n) = \{1, 2, 3, 4, 5\} \quad h(n) = \{6, 7, 8\}$$

$$x(0) = 1$$

$$h(0) = 6$$

$$x(1) = 2$$

$$h(1) = 7$$

$$x(2) = 3$$

$$h(2) = 8$$

$$x(3) = 4$$

$$x(4) = 5$$

$$\text{Total No. of Samples} = M + N - 1 \\ = 5 + 3 - 1$$

$$y(n) = 7 \text{ samples } [n \text{ varies } 0 \text{ to } 6]$$

$$n=0$$

$$y(0) = \sum_{k=0}^4 x(k) h(n-k)$$

$$= x(0) h(0) + x(1) h(-1) + x(2) h(-2) + x(3) h(-3) + x(4) h(-4)$$

$$= 6$$

$$n=1$$

$$y(1) = \sum_{k=0}^4 x(k) h(1-k)$$

$$= x(0) h(1) + x(1) h(0) + x(2) h(-1) + x(3) h(-2) + x(4) h(-3)$$

$$= 7 + 12 = 19$$

$$n=2$$

$$y(2) = \sum_{k=0}^4 x(k) h(2-k)$$

$$= x(0) h(2) + x(1) h(1) + x(2) h(0) + x(3) h(-1) + x(4) h(-2)$$

$$= 8 + 14 + 18$$

$$= 40$$



$$n=3$$

$$y(3) = \sum_{k=0}^4 x(k) h(3-k)$$

$$\begin{aligned} &= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) + x(4)h(-1) \\ &= 16 + 21 + 24 \\ &= 61 \end{aligned}$$

$$n=4$$

$$y(4) = \sum_{k=0}^4 x(k) h(4-k)$$

$$\begin{aligned} &= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) \\ &= 24 + 28 + 30 \\ &= 82 \end{aligned}$$

$$n=5$$

$$y(5) = \sum_{k=0}^4 x(k) h(5-k)$$

$$\begin{aligned} &= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) \\ &= 32 + 35 \\ &= 67 \end{aligned}$$

$$n=6$$

$$y(6) = \sum_{k=0}^4 x(k) h(6-k)$$

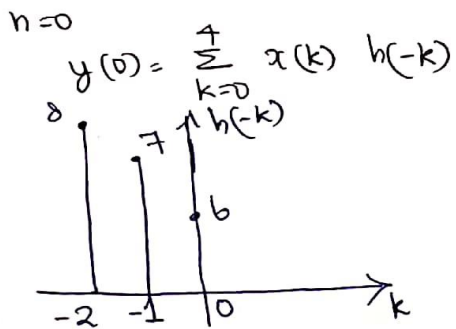
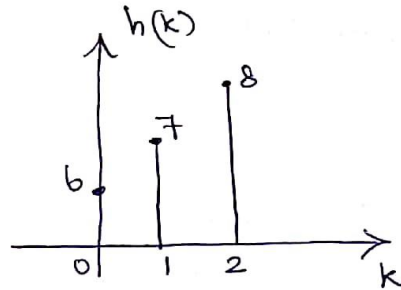
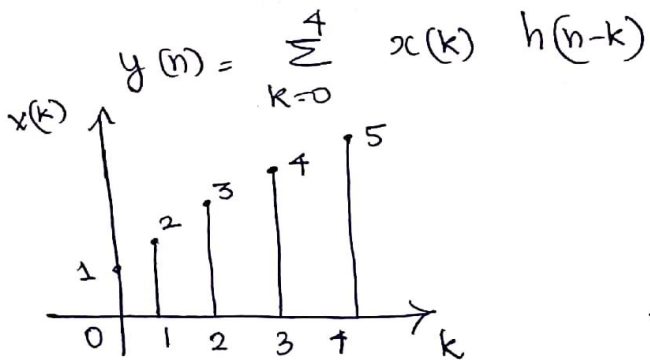
$$\begin{aligned} &= x(0)h(6) + x(1)h(5) + x(2)h(4) + x(3)h(3) + x(4)h(2) \\ &= 40 \end{aligned}$$

$$y(n) = \{ 6, 19, 40, 61, 82, 67, 40 \}$$

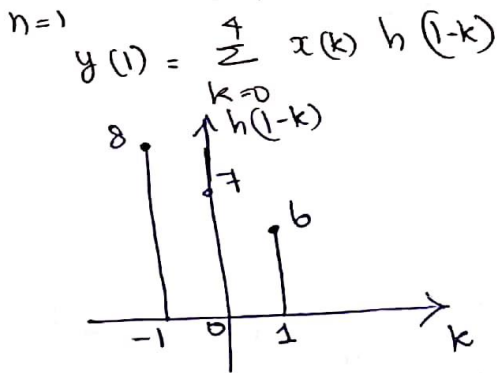


Graphical Method :-

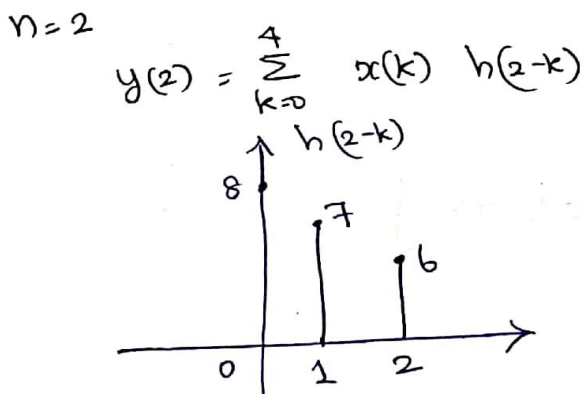
$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$



$$y(0) = 1 \times b = b$$



$$y(1) = 7 + 12 = 19$$

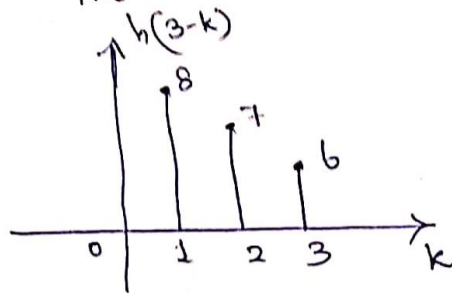


$$y(2) = 8 + 14 + 18 = 40$$



$$n=3$$

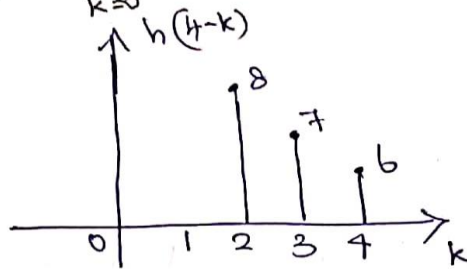
$$y(3) = \sum_{k=0}^3 x(k) h(3-k)$$



$$y(3) = 16 + 21 + 24 = 61$$

$$n=4$$

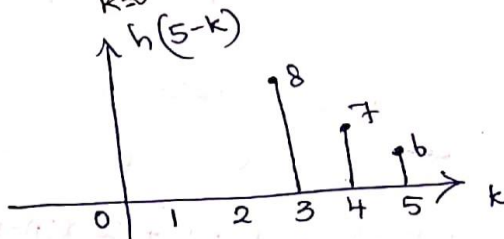
$$y(4) = \sum_{k=0}^4 x(k) h(4-k)$$



$$y(4) = 24 + 28 + 30 = 82$$

$$n=5$$

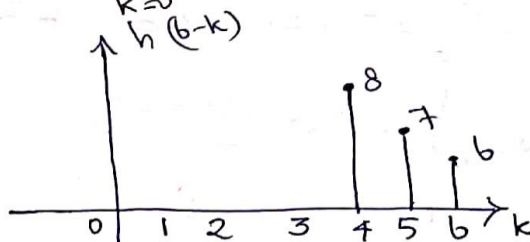
$$y(5) = \sum_{k=0}^5 x(k) h(5-k)$$



$$y(5) = 32 + 35 = 67$$

$$n=6$$

$$y(6) = \sum_{k=0}^6 x(k) h(6-k)$$



$$y(6) = 40$$

$$y(n) = \{ 6, 19, 40, 61, 82, 67, 40 \}$$