



1) Determine the Fourier transform of $x(n) = a^n u(n)$

$$\begin{aligned}
 X(\omega) &= \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n} \\
 &= \sum_{n=-\infty}^{\infty} a^n u(n) e^{j\omega n} \\
 &= \sum_{n=0}^{\infty} a^n e^{j\omega n} \Rightarrow \sum_{n=0}^{\infty} (a e^{j\omega})^n \\
 &= 1 + a e^{j\omega} + (a e^{j\omega})^2 + \dots \infty \quad \left[1 + a + a^2 + \dots = \frac{1}{1-a} \right]
 \end{aligned}$$

$$X(\omega) = \frac{1}{1 - a e^{j\omega}}$$

2) Find DTFT of $x(n) = u(n)$

$$\begin{aligned}
 X(\omega) &= \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n} \\
 &= \sum_{n=-\infty}^{\infty} u(n) e^{j\omega n} \\
 &= \sum_{n=0}^{\infty} e^{j\omega n} \\
 &= 1 + e^{j\omega} + (e^{j\omega})^2 + \dots
 \end{aligned}$$

$$X(\omega) = \frac{1}{1 - e^{j\omega}}$$

3) Find the Fourier transform of $x(n) = \delta(n)$

$$\begin{aligned}
 X(\omega) &= \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n} \\
 &= \sum_{n=-\infty}^{\infty} \delta(n) e^{j\omega n}
 \end{aligned}$$

$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

$$X(\omega) = 1$$



Find the DTFT of $x(n) = \delta(n-n_0)$

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n} \\ &= \sum_{n=-\infty}^{\infty} \delta(n-n_0) e^{j\omega n} \end{aligned}$$

$$n-n_0 = m, \quad n = m+n_0$$

$$= \sum_{m=-\infty}^{\infty} \delta(m) e^{j\omega(m+n_0)}$$

$$= \sum_{m=-\infty}^{\infty} \delta(m) e^{j\omega m} e^{j\omega n_0}$$

$$= \underbrace{\sum_{m=-\infty}^{\infty} \delta(m) e^{j\omega m}}_{1} e^{j\omega n_0}$$

$$X(\omega) = e^{j\omega n_0}$$

5) Find the DTFT of discrete time rectangular pulse of amplitude A and length L .

$$x(n) = \begin{cases} A, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{k=M_1}^{M_2} a^k = \frac{a^{N_1} - a^{N_2+1}}{1-a}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n}$$

$$= \sum_{n=0}^{L-1} A e^{j\omega n}$$

$$= A \left[\frac{(e^{j\omega})^0 - e^{j\omega L}}{1 - e^{j\omega}} \right]$$

$$= A \left[\frac{1 - e^{j\omega L}}{1 - e^{j\omega}} \right]$$



$$= x \left[\frac{e^{j\omega t/2} e^{-j\omega t/2} - e^{-j\omega t/2} e^{j\omega t/2}}{e^{j\omega/2} e^{-j\omega/2} - e^{-j\omega/2} e^{j\omega/2}} \right]$$

$$= x \left[\frac{e^{j\omega t/2} (2j \sin \omega t/2)}{e^{j\omega/2} (2j \sin \omega/2)} \right]$$

$$= x \left[e^{-j\omega t/2} e^{j\omega/2} \left[\frac{\sin \omega t/2}{\sin \omega/2} \right] \right]$$

$$= x \left[e^{j\omega/2 (t-1)} \left[\frac{\sin \omega t/2}{\sin \omega/2} \right] \right]$$