Z - Transform :-

z- transform is used for analysis of discrete time signals & systems

Definition :-

z-transform of $[x(n)]$ is defined as

$$z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

here, z is a complex Variable

$$z = r \cdot e^{j\omega}$$

r is the magnitude of z.

ω is the phase angle of z $\omega = \angle z$

Inverse z- transform :-

$$x(n) = \frac{1}{2\pi j} \oint_C X(v) v^{n-1} dv.$$

Types of z-Transform :-

(i) Bilateral (or) two sided z-transform.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

(ii) Unilateral (or) one sided z-transform

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

Roc [Region of convergence] :-

Roc is the region where z-transform converges.

Significance :-

* Roc given an idea about values of z for which z-transform can be calculated.
* Roc can be used to determine causality and stability of the system.

Relationship between DTFT and z-transform :-

DTFT :- $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

z-transform :- $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

z is a complex Variable, $z = re^{j\omega}$

$$z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n)(re^{j\omega})^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, r^{-n}\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[x(n)\, r^{-n}\right] e^{-j\omega n}$$

$$Z[x(n)] = DTFT[r^{-n} x(n)]$$

$$|z| = r = 1$$

$$Z[x(n)] = DTFT[x(n)]$$

$$X(z) = X(\omega) \Big/ z = e^{j\omega}.$$