Power Series Expansion (or) Long Division Method :-

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \{ \dots + x(-2) z^{2} + x(-1) z^{1} + x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2} + \dots \}$$

$$x(n) = \{ \dots, x(-2), x(-1), x(0), x(1), x(2) \}$$

$x(n)$ can be obtained by collecting the co-efficients of z in $X(z)$ expansion

NOTE :-

1) when the ROC is $|z| > |a|$ (causal s/m) then expand $X(z)$ such that the powers of z are negative

2) when the ROC is $|z| < |a|$ (Non-causal s/m) then expand $X(z)$ such that the powers of z are positive.

① Find the inverse z-transform using power series expansion method :-

$$X(z) = \frac{1}{1-az^{-1}} \qquad \text{ROC} : |z| > |a|$$

$$
\begin{array}{r|l}
 & 1 + az^{-1} + a^2 z^{-2} \\
\hline
1-az^{-1} & \cancel{1} \\
 & \cancel{1} - az^{-1} \\
 & (-) \quad (+) \\
\hline
 & a\cancel{z^{-1}} \\
 & + a\cancel{z^{-1}} - a^2 z^{-2} \\
 & (-) \qquad (+) \\
\hline
 & a^2 \cancel{z^{-2}} \\
 & a^2 \cancel{z^{-2}} - a^3 z^{-3} \\
 & (-) \qquad (+) \\
\hline
 & a^3 z^{-3}
\end{array}
$$

$$X(z) = \{ 1 + az^{-1} + a^2 z^{-2} + \dots \}$$

$$x(n) = \{ 1 + a + a^2 + a^3 + \dots \}$$

$\therefore$ causal system

(ii) $$X(z) = \frac{1}{1 - az^{-1}} \quad ; \quad ROC : |z| < |a|$$

$$-a^{-1}z - a^{-2}z^2 - a^{-3}z^3$$

$-az^{-1} + 1 \;\big)\; \cancel{1}$

$\cancel{1} - a^{-1}z$

$(-) \; (+)$

$\cancel{a^{-1}z}$

$\cancel{a^{-1}z} - a^{-2}z^2$

$(-) \quad (+)$

$\cancel{a^{-2}z^2}$

$(-)\ \cancel{a^{-2}z^2} - a^{-3}z^3$

$(+)$

$a^{-3}z^3$

$$X(z) = \{ \dots\dots - a^{-3}z^3 - a^{-2}z^2 - a^{-1}z \}$$

$$x(n) = \{ \dots\dots -a^{-3}, -a^{-2}, -a^{-1} \}$$

Non-causal system $\qquad x(n) = -a^n u(-n-1)$

HW

2) $$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$ when $x(n)$ is causal & Non-causal

$x(n) = \{ 1, 4, 7, 10 \dots \} \rightarrow$ causal

$x(n) = \{ + \dots 11, 8, 5, 2, \underset{\uparrow}{0} \} \rightarrow$ Non-causal

3) $X(z) = \dfrac{z+1}{z^2-3z+2}$ when $x(n)$ is causal & Non-causal

(i)

$$z^{-1} + 4z^{-2} + 10z^{-3} + 22z^{-4} + \dots$$

z^2-3z+2)

$\cancel{z} + 1$

$\cancel{z} - 3 + 2z^{-1}$

(−) (+) (−)

$\cancel{4} - 2z^{-1}$

$\cancel{4} - 12z^{-1} + 8z^{-2}$

(−) (+) (−)

$\cancel{10z^{-1}} - 8z^{-2}$

(−) $\cancel{10z^{-1}} - 30z^{-2} + 20z^{-3}$

(+) (−)

$22z^{-2} - 20z^{-3}$

$$X(z) = \{ z^{-1} + 4z^{-2} + 10z^{-3} + 22z^{-4} + \dots \}$$

$$x(n) = \{ 0, 1, 4, 10, 22, \dots \}$$

(ii) Non-causal :-

$$\tfrac{1}{2} + \tfrac{5}{4}z + \tfrac{13}{8}z^2$$

$2-3z+z^2$)

$\cancel{1} + z$

$\cancel{1} - \tfrac{3}{2}z + \tfrac{1}{2}z^2$

(−) (+) (−)

$\cancel{\tfrac{5}{2}z} - \tfrac{1}{2}z^2$

$\cancel{\tfrac{5}{2}z} - \tfrac{15}{4}z^2 + \tfrac{5}{4}z^3$

(−) (+) (−)

$\cancel{\tfrac{13}{4}z^2} - \tfrac{5}{4}z^3$

$\cancel{\tfrac{13}{4}z^2} - \tfrac{39}{8}z^3 + \tfrac{13}{8}z^4$

(−) (+) (−)

$\tfrac{29}{8}z^3 - \tfrac{13}{8}z^4$

$$X(z) = \{ + \dots \tfrac{13}{8}z^2 + \tfrac{5}{4}z + \tfrac{1}{2} \}$$

$$x(n) = \{ + \dots \tfrac{13}{8}, \tfrac{5}{4}, \tfrac{1}{2}, 0 \}$$