# LTI DT System Analysis using Z Transform

Solving difference equation using z-transform :-

Shifting property of Unilateral z-transform.

$$y(n-2) \overset{ZT}{\longleftrightarrow} z^{-2}\,Y(z) + z^{-1}\,y(-1) + z^{0}\,y(-2)$$

$$y(n-3) \overset{ZT}{\longleftrightarrow} z^{-3}\,Y(z) + z^{-2}\,y(-1) + z^{-1}\,y(-2) + z^{0}\,y(-3)$$

1) Given that $y(-1)=5$, $y(-2)=0$. Solve the difference equation $y(n) - 3y(n-1) - 4y(n-2) = 0$

$$Y(z) - 3\left[z^{-1}Y(z) + z^{0}y(-1)\right] - 4\left[z^{-2}Y(z) + z^{-1}y(-1) + z^{0}y(-2)\right] = 0$$

$$Y(z) - 3\left[z^{-1}Y(z) + 5z^{0}\right] - 4\left[z^{-2}Y(z) + 5z^{-1}\right] = 0$$

$$Y(z) - 3z^{-1}Y(z) - 15z^{0} - 4z^{-2}Y(z) - 20z^{-1} = 0$$

$$Y(z)\left[1 - 3z^{-1} - 4z^{-2}\right] = 15z^{0} + 20z^{-1}$$

$$Y(z) = \frac{15z^{0} + 20z^{-1}}{1 - 3z^{-1} - 4z^{-2}}$$

Multiply and divide by $z^2$

$$Y(z) = \frac{z^2}{z^2}\,\frac{15z^{0} + 20z^{-1}}{1 - 3z^{-1} - 4z^{-2}}$$

$$Y(z) = \frac{15z^2 + 20z}{z^2 - 3z - 4}$$

$$\frac{Y(z)}{z} = \frac{15z + 20}{z^2 - 3z - 4}$$

$$\frac{Y(z)}{z} = \frac{15z + 20}{(z-4)(z+1)}$$

$$\frac{15z+20}{(z-4)(z+1)} = \frac{A}{z-4} + \frac{B}{z+1}$$

$$15z+20 = A(z+1) + B(z-4)$$

$z=-1$

$5 = B(-5)$

$\boxed{B=-1}$

$z=4$

$80 = A(5)$

$\boxed{A=16}$

$$\frac{H(z)}{z} = \frac{16}{z-4} - \frac{1}{z+1}$$

$$H(z) = 16\frac{z}{z-4} - \frac{z}{z+1}$$

$$h(n) = 16(4)^n u(n) - (-1)^n u(n)$$

② Find the output of the system whose input output relate by $y(n) = 7y(n-1) - 12y(n-2) + 2x(n) - x(n-2)$ for input $x(n) = u(n)$

Taking z transform on both sides

$$Y(z) = 7z^{-1}Y(z) - 12z^{-2}Y(z) + 2X(z) - z^{-2}X(z)$$

$$Y(z) - 7z^{-1}Y(z) + 12z^{-2}Y(z) = 2X(z) - z^{-2}X(z)$$

$$Y(z)\left[1 - 7z^{-1} + 12z^{-2}\right] = X(z)\left[2 - z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-2}}{1 - 7z^{-1} + 12z^{-2}}$$

Multiply & Divide by $z^2$

$$H(z) = \frac{2z^2 - 1}{z^2 - 7z + 12}$$

$x(n) = u(n)$

$X(z) = \frac{z}{z-1}$

$$Y(z) = \frac{z(2z^2 - 1)}{(z-1)(z^2 - 7z + 12)}$$

$$\frac{Y(z)}{z} = \frac{2z^2 - 1}{(z-1)(z-3)(z-4)}$$

$$\frac{2z^2 - 1}{(z-1)(z-3)(z-4)} = \frac{A}{z-1} + \frac{B}{z-3} + \frac{C}{z-4}$$

$$2z^2 - 1 = A(z-3)(z-4) + B(z-1)(z-4) + C(z-1)(z-3)$$

Put $z = 1$

$1 = A(-2)(-3) + B(0) + C(0)$

$\boxed{A = 1/6}$

Put $z = 3$

$17 = A(0) + B(2)(-1) + C(0)$

$\boxed{B = -17/2}$

Put $z = 4$

$31 = A(0) + B(0) + C(3)(1)$

$\boxed{C = 31/3}$

$$\frac{Y(z)}{z} = \frac{1}{6}\left(\frac{1}{z-1}\right) - \frac{17}{2}\left(\frac{1}{z-3}\right) + \frac{31}{3}\left(\frac{1}{z-4}\right)$$

$$Y(z) = \frac{1}{6}\left(\frac{z}{z-1}\right) - \frac{17}{2}\left(\frac{z}{z-3}\right) + \frac{31}{3}\left(\frac{z}{z-4}\right)$$

$$y(n) = \frac{1}{6}(1)^n u(n) - \frac{17}{2}(3)^n u(n) + \frac{31}{3}(4)^n u(n)$$

$$\therefore y(n) = \frac{1}{6}u(n) - \frac{17}{2}(3)^n u(n) + \frac{31}{3}(4)^n u(n)$$