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DEPARTMENT OF AEROSPACE ENGINEERING

PRINCIPAL STRESSES AND STRAINS

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3.2. PRINCIPAL PLANES AND PRINCIPAL STRESSES

The planes, which have no shear stress, are known as principal planes. Hence principal planes are the planes of zero shear stress. These planes carry only normal stresses.

The normal stresses, acting on a principal plane, are known as principal stresses.

3.3. METHODS FOR DETERMINING STRESSES ON OBLIQUE SECTION

The stresses on oblique section are determined by the following methods :

1. Analytical method, and
2. Graphical method.

3.4. ANALYTICAL METHOD FOR DETERMINING STRESSES ON OBLIQUE SECTION

The following two cases will be considered :

1. A member subjected to a direct stress in one plane.
2. The member is subjected to like direct stresses in two mutually perpendicular directions.

3.4.1. A Member Subjected to a Direct Stress in one Plane. Fig. 3.1 (a) shows a rectangular member of uniform cross-sectional area A and of unit thickness.

Let P = Axial force acting on the member.

A = Area of cross-section, which is perpendicular to the line of action of the force P.

The stress along x-axis, $\sigma = \dfrac{P}{A}$

Hence, the member is subjected to a stress along x-axis.

Consider a cross-section EF which is perpendicular to the line of action of the force P.

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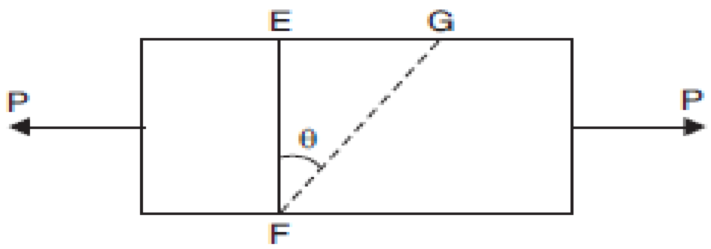

Fig. 3.1 (*a*)

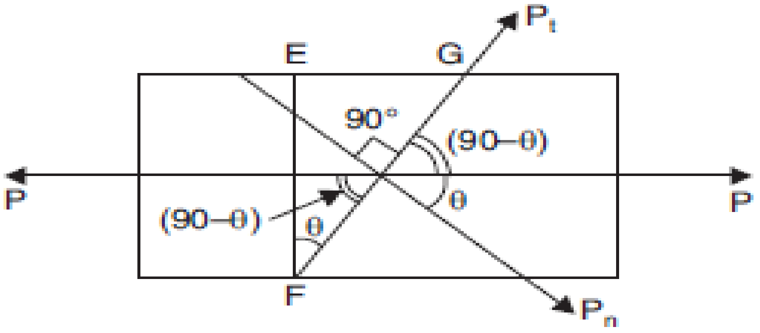

Fig. 3.1 (*b*)

Then area of section, $EF = EF \times 1 = A$.

The stress on the section EF is given by

$$\sigma = \frac{\text{Force}}{\text{Area of } EF} = \frac{P}{A} \qquad \ldots(i)$$

The stress on the section EF is entirely normal stress. There is no shear stress (or tangential stress) on the section EF.

Now consider a section FG at an angle θ with the normal cross-section EF as shown in Fig. 3.1 (*a*).

Area of section $FG = FG \times 1$ (member is having unit thickness)

$$= \frac{EF}{\cos\theta} \times 1 \qquad \left(\because \text{In } \Delta\, EFG,\ \frac{EF}{FG} = \cos\theta \ \therefore\ FG = \frac{EF}{\cos\theta}\right)$$

$$= \frac{A}{\cos\theta} \qquad (\because \quad EF \times 1 = A)$$

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$\therefore$ Stress on the section, FG

$$= \frac{\text{Force}}{\text{Area of section } FG} = \frac{P}{\left(\frac{A}{\cos\theta}\right)} = \frac{P}{A}\cos\theta$$

$$= \sigma\cos\theta \qquad \left(\because \frac{P}{A} = \sigma\right) \qquad \ldots(3.1)$$

This stress, on the section FG, is parallel to the axis of the member (*i.e.*, this stress is along x-axis). This stress may be resolved in two components. One component will be normal to the section FG whereas the second component will be along the section FG (*i.e.*, tangential to the section FG). The normal stress and tangential stress (*i.e.*, shear stress) on the section FG are obtained as given below [Refer to Fig. 3.1 (*b*)].

Let P_n = The component of the force P, normal to section FG

= $P\cos\theta$

P_t = The component of force P, along the surface of the section FG (or tangential to the surface FG)

= $P\sin\theta$

σ_n = Normal stress across the section FG

σ_t = Tangential stress (*i.e.*, shear stress) across the section FG.

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∴ Normal stress and tangential stress across the section FG are obtained as,

Normal stress, $$\sigma_n = \frac{\text{Force normal to section } FG}{\text{Area of section } FG}$$

$$= \frac{P_n}{\left(\dfrac{A}{\cos\theta}\right)} = \frac{P\cos\theta}{\left(\dfrac{A}{\cos\theta}\right)} \qquad (\because \; P_n = P\cos\theta)$$

$$= \frac{P}{A}\cos\theta \,.\, \cos\theta = \frac{P}{A}\cos^2\theta$$

$$= \sigma\cos^2\theta \qquad \left(\because \; \frac{P}{A} = \sigma\right) \qquad \ldots(3.2)$$

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Tangential stress (*i.e.*, shear stress),

$$\sigma_t = \frac{\text{Tangential force across section } FG}{\text{Area of section } FG}$$

$$= \frac{P_t}{\left(\frac{A}{\cos\theta}\right)} = \frac{P\sin\theta}{\left(\frac{A}{\cos\theta}\right)} \qquad (\because \quad P_t = P\sin\theta)$$

$$= \frac{P}{A}\sin\theta \,.\, \cos\theta = \sigma\sin\theta.\,\cos\theta$$

$$= \frac{\sigma}{2} \times 2\sin\theta\cos\theta \qquad \text{[Multiplying and dividing by 2]}$$

$$= \frac{\sigma}{2}\sin 2\theta \qquad (\because \quad 2\sin\theta\cos\theta = \sin 2\theta) \qquad ...(3.3)$$

From equation (3.2), it is seen that the normal stress (σ_n) on the section *FG* will be maximum, when $\cos^2 \theta$ or $\cos \theta$ is maximum. And $\cos \theta$ will be maximum when $\theta = 0°$ as $\cos 0° = 1$. But when $\theta = 0°$, the section *FG* will coincide with section *EF*. But the section *EF* is normal to the line of action of the loading. This means the plane normal to the axis of loading will carry the maximum normal stress.

$$\therefore \text{ Maximum normal stress, } = \sigma \cos^2 \theta = \sigma \cos^2 0° = \sigma \quad \text{...(3.4)}$$

From equation (3.3), it is observed that the tangential stress (*i.e.*, shear stress) across the section *FG* will be maximum when $\sin 2\theta$ is maximum. And $\sin 2\theta$ will be maximum when $\sin 2\theta = 1$ or $2\theta = 90°$ or $270°$

or $$\theta = 45° \text{ or } 135°.$$

This means the shear stress will be maximum on two planes inclined at 45° and 135° to the normal section *EF* as shown in Figs. 3.1 (*c*) and 3.1 (*d*).

$$\therefore \quad \text{Max. value of shear stress} = \frac{\sigma}{2} \sin 2\theta = \frac{\sigma}{2} \sin 90° = \frac{\sigma}{2}. \quad \text{...(3.5)}$$

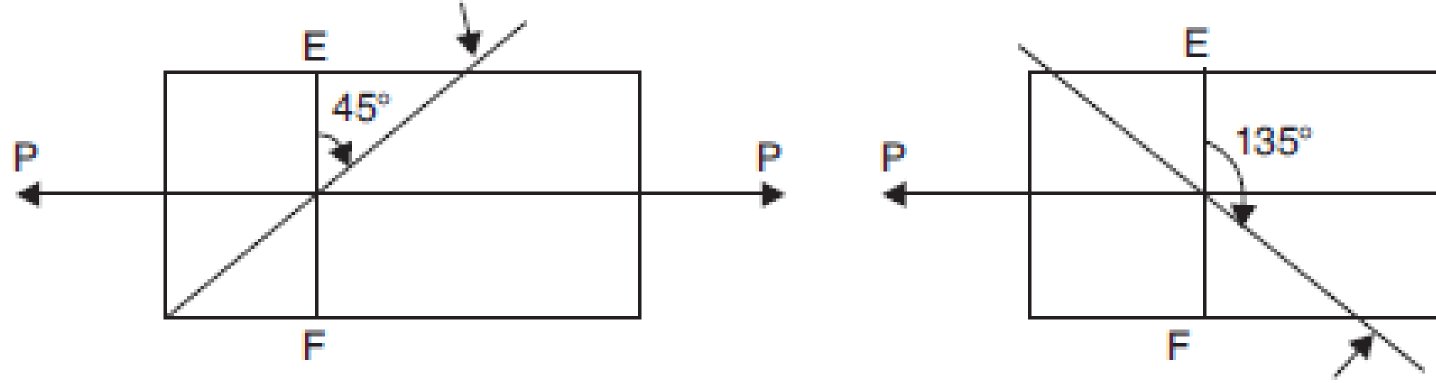

Fig. 3.1 (*c*) Fig. 3.1 (*d*)

From equations (3.4) and (3.5) it is seen that maximum normal stress is equal to σ whereas the maximum shear stress is equal to $\sigma/2$ or equal to half the value of greatest normal stress.

Problem 3.1. *A rectangular bar of cross-sectional area 10000 mm² is subjected to an axial load of 20 kN. Determine the normal and shear stresses on a section which is inclined at an angle of 30° with normal cross-section of the bar.*

Sol. Given :

Cross-sectional area of the rectangular bar,

$$A = 10000 \text{ mm}^2$$

Axial load, $P = 20 \text{ kN} = 20{,}000 \text{ N}$

Angle of oblique plane with the normal cross-section of the bar,

$$\theta = 30^\circ$$

Now direct stress $\sigma = \dfrac{P}{A} = \dfrac{20000}{10000} = 2 \text{ N/mm}^2$

Let σ_n = Normal stress on the oblique plane

σ_t = Shear stress on the oblique plane.

Using equation (3.2) for normal stress, we get

$$\sigma_n = \sigma \cos^2 \theta$$

$$= 2 \times \cos^2 30^\circ \qquad (\because \sigma = 2 \text{ N/mm}^2)$$

$$= 2 \times 0.866^2 \qquad (\because \cos 30^\circ = 0.866)$$

$$= \mathbf{1.5 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Using equation (3.3) for shear stress, we get

$$\sigma_t = \frac{\sigma}{2} \sin 2\theta = \frac{2}{2} \times \sin (2 \times 30^\circ)$$

$$= 1 \times \sin 60^\circ = \mathbf{0.866 \text{ N/mm}^2}. \quad \textbf{Ans.}$$

Problem 3.3. *A rectangular bar of cross-sectional area of 11000 mm² is subjected to a tensile load P as shown in Fig. 3.3. The permissible normal and shear stresses on the oblique plane BC are given as 7 N/mm² and 3.5 N/mm² respectively. Determine the safe value of P.*

Sol. Given :

Area of cross-section, $A = 11000 \text{ mm}^2$

Normal stress, $\sigma_n = 7 \text{ N/mm}^2$

Shear stress, $\sigma_t = 3.5 \text{ N/mm}^2$

Angle of oblique plane with the axis of bar = 60°.

∴ Angle of oblique plane BC with the normal cross-section of the bar,

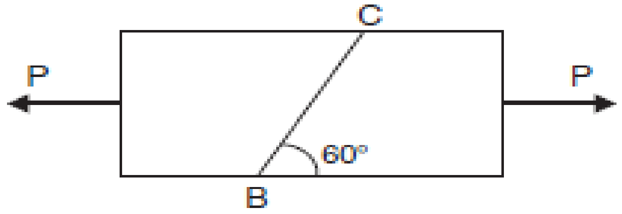

Fig. 3.3

$$\theta = 90° - 60° = 30°$$

Let P = Safe value of axial pull

σ = Safe stress in the member.

Using equation (3.2),

$$\sigma_n = \sigma \cos^2 \theta \quad \text{or} \quad 7 = \sigma \cos^2 30°$$

$$= \sigma (0.866)^2. \qquad (\because \cos 30° = 0.866)$$

$$\therefore \quad \sigma = \frac{7}{0.866 \times 0.866} = 9.334 \text{ N/mm}^2$$

Using equation (3.3),

$$\sigma_t = \frac{\sigma}{2} \sin 2\theta$$

or

$$3.5 = \frac{\sigma}{2} \sin 2 \times 30° = \frac{\sigma}{2} \sin 60° = \frac{\sigma}{2} \times 0.866$$

$$\therefore \quad \sigma = \frac{3.5 \times 2}{0.866} = 8.083 \text{ N/mm}^2.$$

The safe stress is the least of the two, *i.e.*, 8.083 N/mm².

∴ Safe value of axial pull,

P = Safe stress × Area of cross-section

$= 8.083 \times 11000 = 88913 \text{ N} = 88.913 \text{ kN}.$ **Ans.**


Thank You