LTI DT System Analysis using Z Transform

① The output y(n) of discrete time LTI system is $2\left(\frac{1}{3}\right)^n u(n)$ when the input signal $x(n) = u(n)$. find the impulse response of the system.

Soln :-

$$Y(z) = 2\left(\frac{z}{z-1/3}\right) \qquad X(z) = \frac{z}{z-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2\left(\frac{z}{z-1/3}\right)}{z/z-1}$$

$$H(z) = \frac{2(z-1)}{z-1/3}$$

Multiply & divide by z

$$H(z) = \frac{2z(z-1)}{z(z-1/3)} \Rightarrow \frac{H(z)}{z} = \frac{2(z-1)}{z(z-1/3)}$$

$$\frac{2(z-1)}{z(z-1/3)} = \frac{A}{z} + \frac{B}{z-1/3}$$

$$2(z-1) = A(z-1/3) + B(z)$$

Put $z=0$ | Put $z = 1/3$

$-2 = A(-1/3)$ | $-4/3 = B(1/3)$

$\boxed{A = 6}$ | $\boxed{B = -4}$

$$\frac{H(z)}{z} = \frac{6}{z} - \frac{4}{z-1/3}$$

$$H(z) = 6(z/z) - 4\left(\frac{z}{z-1/3}\right)$$

$$h(n) = 6\,\delta(n) - 4\left(\frac{1}{3}\right)^n u(n)$$

② compute the response of the system $y(n) = 0.7y(n-1) - 0.12\,y(n-2) + x(n-1) + x(n-2)$ to the i/p $x(n) = n\,u(n)$ is system stable.

$$y(n) = 0.7y(n-1) - 0.12\,y(n-2) + x(n-1) + x(n-2)$$

Taking z-transform on both sides

$$Y(z) = 0.7\,z^{-1}\,Y(z) - 0.12\,z^{-2}\,Y(z) + X(z)\,z^{-1} + z^{-2}X(z)$$

$$Y(z) - z^{-1}\,0.7\,Y(z) + 0.12\,z^{-2}\,Y(z) = z^{-1}X(z) + z^{-2}X(z)$$

$$Y(z)\left[1 - 0.7z^{-1} + 0.12z^{-2}\right] = X(z)\left[z^{-1} + z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} + z^{-2}}{1 - 0.7z^{-1} + 0.12z^{-2}}$$

Multiply & Divide by z^2

$$H(z) = \frac{z+1}{z^2 - 0.7z + 0.12}$$

$$= \frac{z+1}{z^2 - 0.3z - 0.4z + 0.12}$$

$$H(z) = \frac{z+1}{(z-0.3)(z-0.4)}$$

$$\frac{Y(z)}{X(z)} = \frac{z+1}{(z-0.3)(z-0.4)}$$

$$Y(z) = X(z) \cdot \frac{z+1}{(z-0.3)(z-0.4)}$$

i/p : $x(n) = n\, u(n)$

$$X(z) = \frac{z}{(z-1)^2}$$

$$Y(z) = \frac{z}{(z-1)^2} \cdot \frac{z+1}{(z-0.3)(z-0.4)}$$

$$Y(z) = \frac{z(z+1)}{(z-1)^2(z-0.3)(z-0.4)} \Rightarrow \frac{Y(z)}{z} = \frac{z+1}{(z-1)^2(z-0.3)(z-0.4)}$$

$$\frac{z+1}{(z-1)^2(z-0.3)(z-0.4)} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{(z-0.3)} + \frac{D}{(z-0.4)}$$

$$z+1 = A(z-1)(z-0.3)(z-0.4) + B(z-0.3)(z-0.4) + C(z-1)^2(z-0.4) + D(z-1)^2(z-0.3)$$

Put $z=1$

$2 = B(0.42)$

$B = 2/0.42$

$B = 4.761$

Put $z = 0.3$

$1.3 = C(-0.049)$

$C = -\frac{1.3}{0.049}$

$C = -26.53$

Put $z = 0.4$

$1.4 = D(0.036)$

$D = \frac{1.4}{0.036}$

$D = 38.88$

Put $z=0$

$1 = A(-0.12) - 0.9834$

$1.4834 = A(-0.12) \Rightarrow \frac{-1.4834}{0.12}$

$A = -12.36$

$$\frac{Y(z)}{z} = \frac{-12.36}{z-1} + \frac{4.761}{(z-1)^2} - \frac{26.53}{z-0.3} + \frac{38.88}{z-0.4}$$

$$Y(z) = -12.36\left(\frac{z}{z-1}\right) + 4.761\left(\frac{z}{(z-1)^2}\right) - 26.53\left(\frac{z}{z-0.3}\right) + 38.88\left(\frac{z}{z-0.4}\right)$$

$$y(n) = -12.36\, u(n) + 4.761\, n\, u(n) - 26.53\,(0.3)^n u(n) + 38.88\,(0.4)^n u(n)$$