# SNS COLLEGE OF TECHNOLOGY

**An Autonomous Institution**

**Coimbatore-35**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

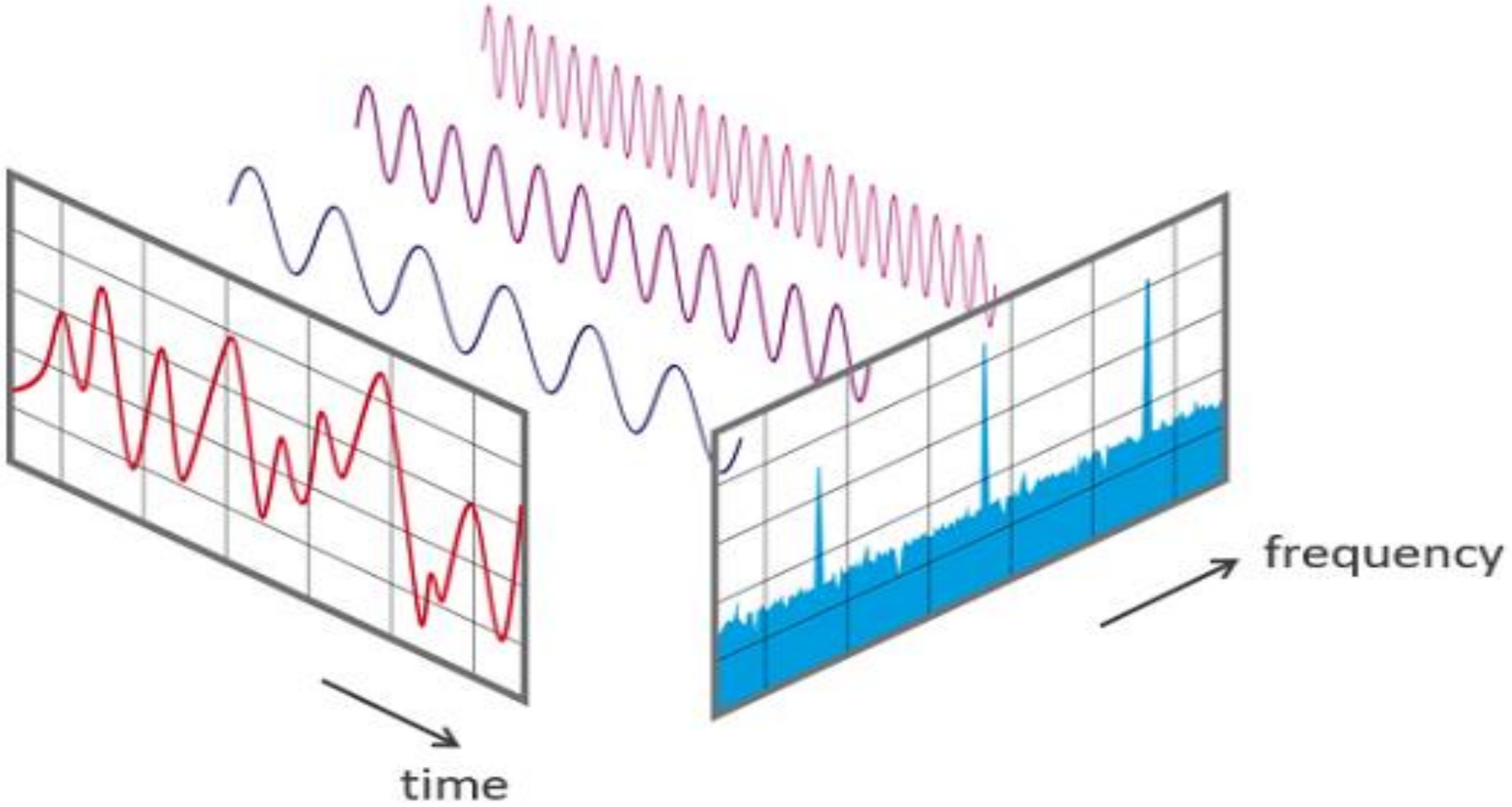
## 23ECT202 – SIGNALS AND SYSTEMS

II YEAR/ III SEMESTER

## UNIT 5 – LTI DISCRETE TIME SYSTEMS

TOPIC – LTI DT SYSTEM ANALYSIS

# DISCRETE TIME FOURIER TRANSFORM



# DISCRETE TIME FOURIER TRANSFORM

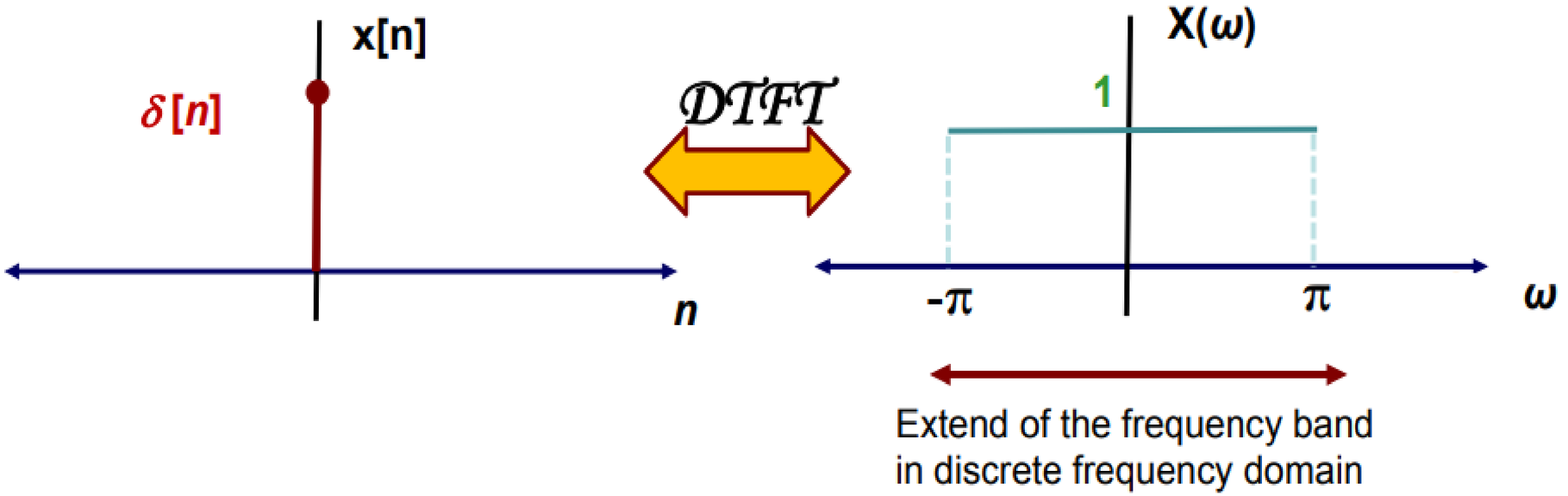
- Discrete time signals are analyzed with the help of periodic Signals.
- DTFS - Periodic Signals
- DTFT - Periodic & Non Periodic Signals
- Discrete Time Fourier Transform describes the spectrum of discrete time signals.
- DTFT of discrete time signal x(n) is given as

$$X(\omega) = \sum_{-\infty}^{\infty} x(n)\, e^{-jwn}$$

# DTFT IMPULSE FUNCTION

$\delta(n) = 1$ for $n = 0$

$= 0$ for $n \neq 0$



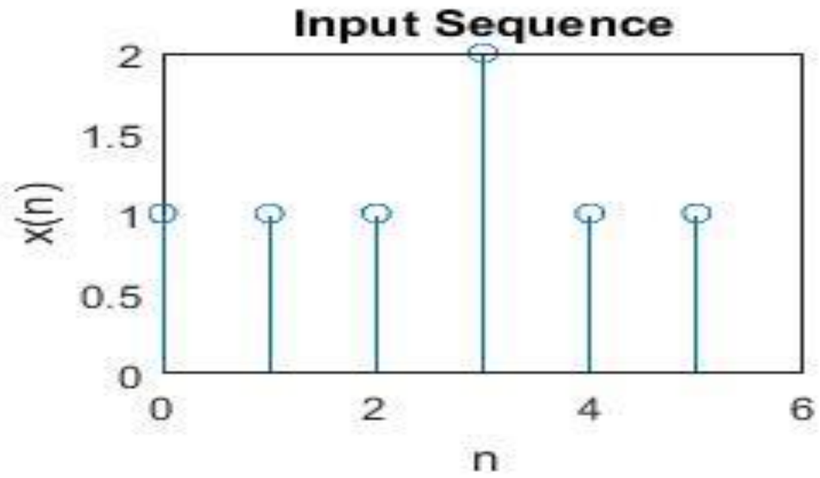
# LINEAR CONVOLUTION

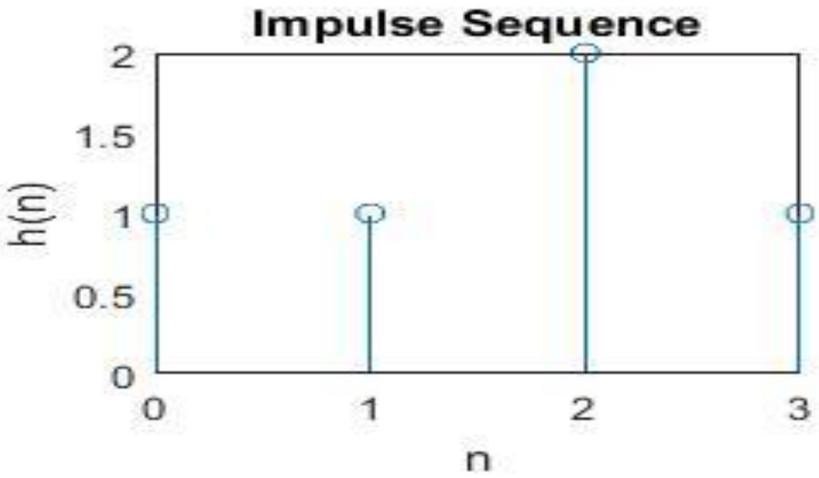
- The behavior of the LTI system is completely characterized by the unit sample response h(n)

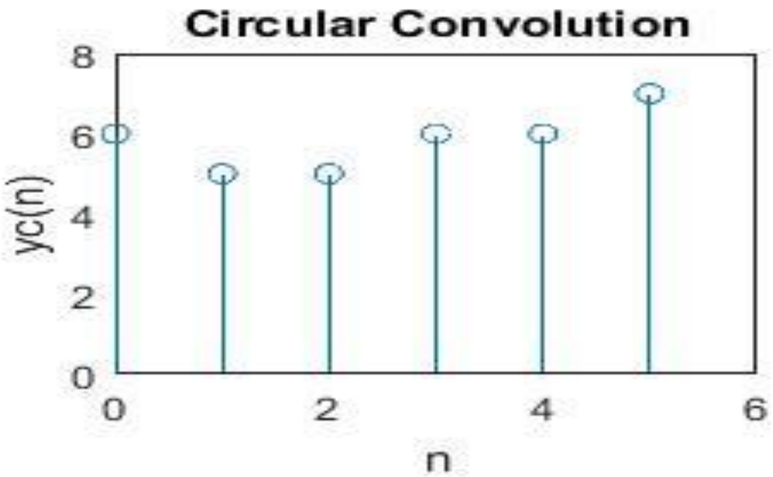
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

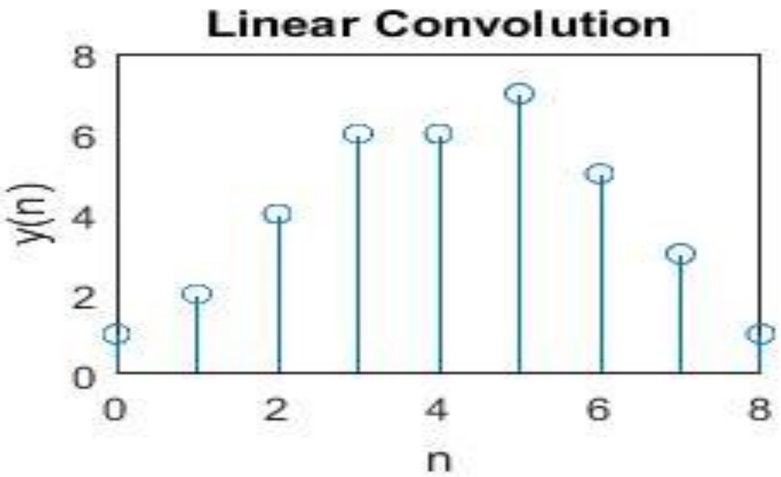
- It is the linear convolution of x(n) and h(n) gives y(n)

# CONVOLUTION SUM



# CONVOLUTION SUM

$x(n) = \{1, 1, 1, 2, 1, 1\}$ $\quad h(n) = \{1, 1, 2, 1\}$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 1 | 1 | 2 | 1 | 1 | | |
| | | | | 1 | 1 | 2 | 1 | | |
| | | 1 | 1 | 1 | 2 | 1 | 1 | | |
| | 2 | 2 | 2 | 4 | 2 | 2 | | | |
| 1 | 1 | 1 | 2 | 1 | 1 | | | | |
| 1 | 2 | 4 | 6 | 6 | 7 | 5 | 3 | 1 | |

$y(n) = \{1, 2, 4, 6, 6, 7, 5, 3, 1\}$

# PROPERTIES OF LINEAR CONVOLUTION

- It can be classified into

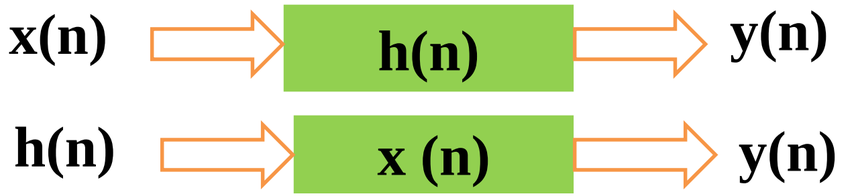
1. Commutative Property

2. Associative Property

3. Distributive Property

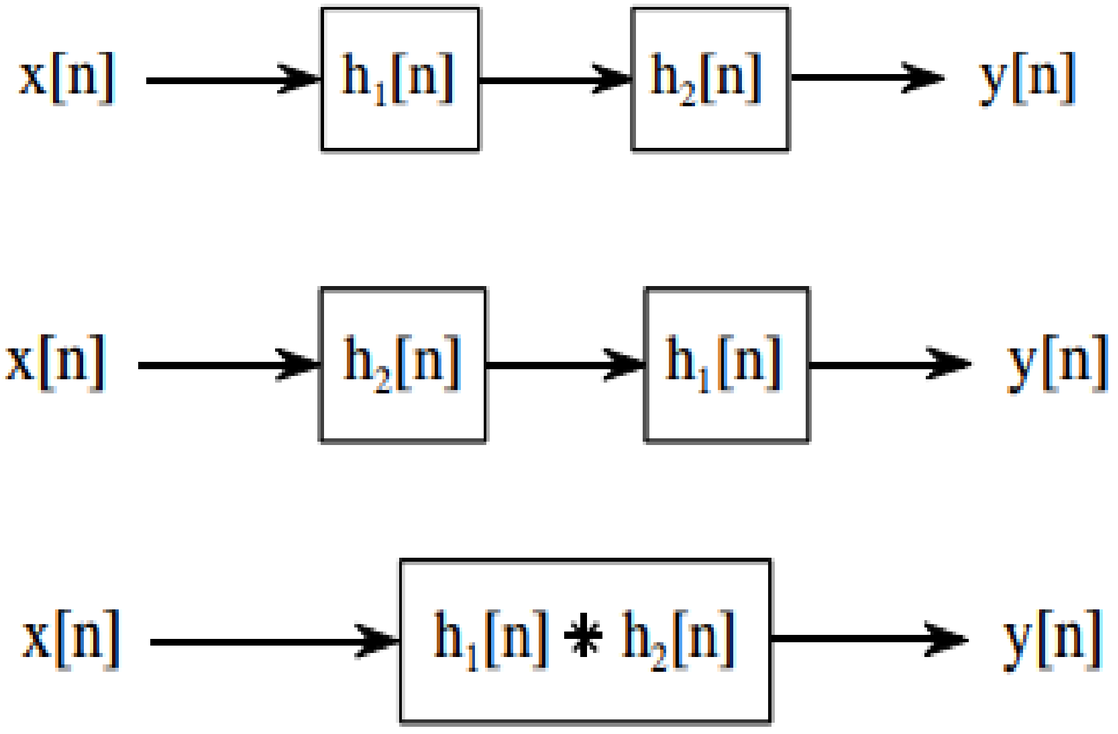
**Commutative :**

$$y(n) = x(n) * h(n) = h(n) * x(n)$$



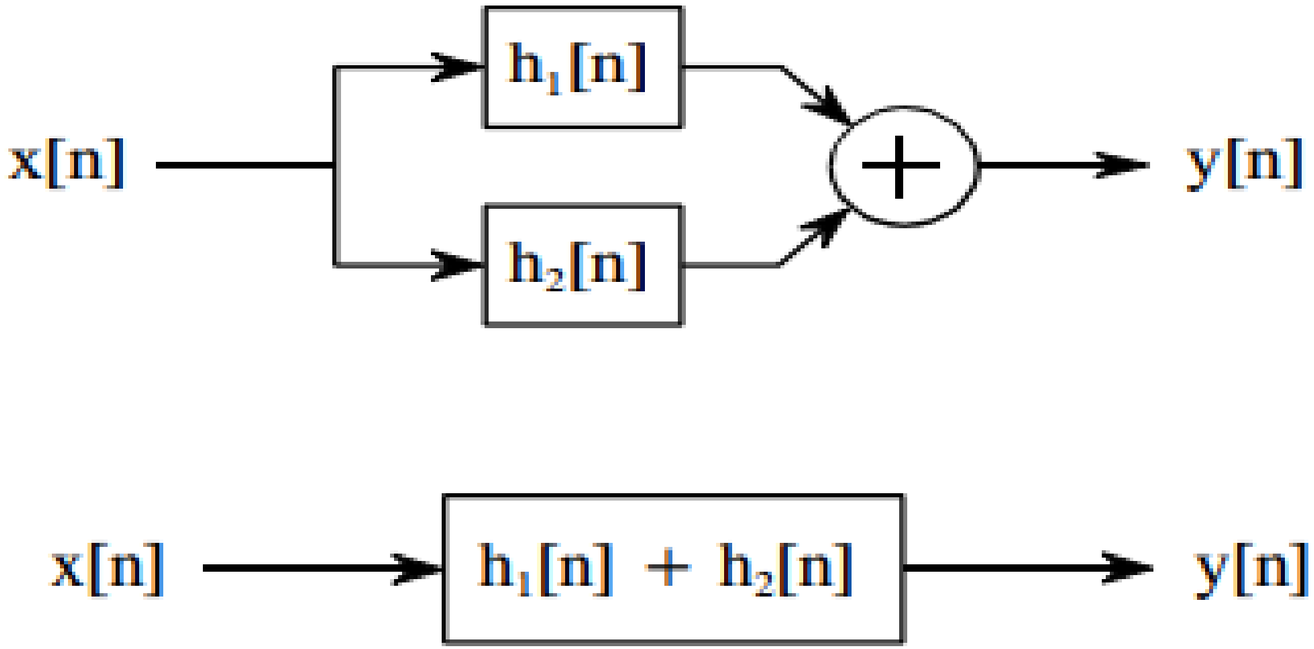
# ASSOCIATIVE PROPERTY

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$



# DISTRIBUTIVE PROPERTY

$$x(n) * h_1(n) + x(n) * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$



# DIFFERENCE EQUATION

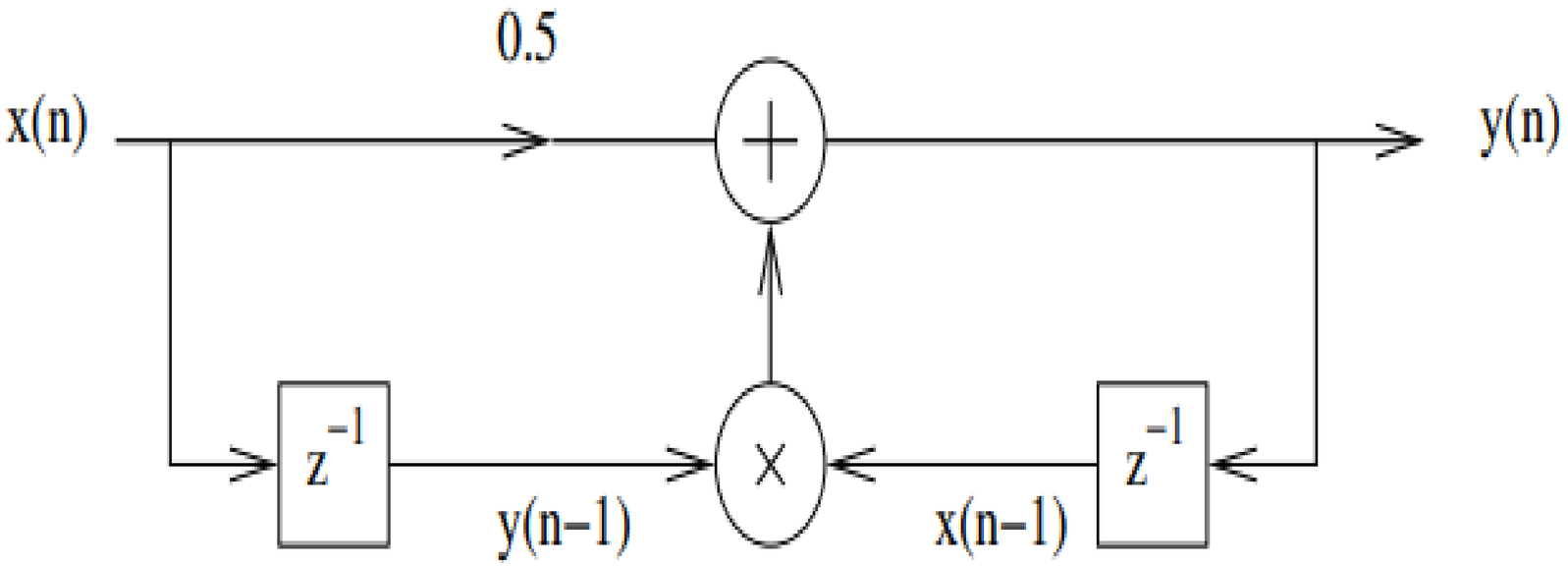
- **Difference Equation:** It is an efficient way to implement discrete time systems
- The convolution of input sequence x(n) and unit sample response h(n) gives the output y(n)

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

- Two types of systems depending upon the length of unit sample response h(k)

# RECURSIVE DIFFERENCE EQUATION

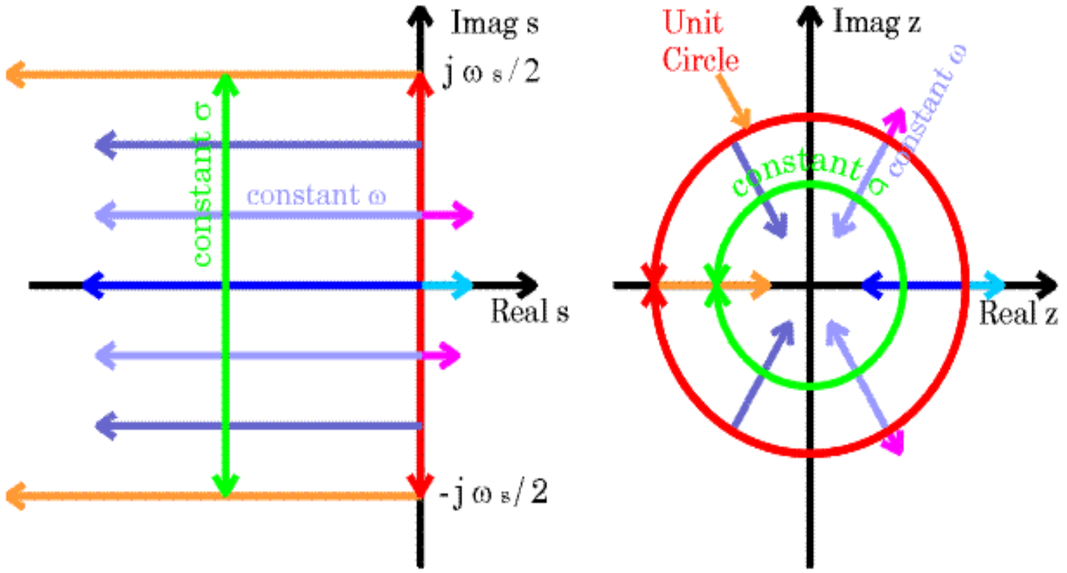
$$y(n) = y(n-1)\, x(n-1) + 0.5\, x(n)$$



# Z TRANSFORM

- Z transform is used for the analysis of discrete time signals.
- It is more broad compared to Discrete Time Fourier Transform
- It is very much useful in discrete time signals as well as system analysis
- x(n) and X(Z) is called Z transform pair

**x(n)** ⟺ **X(Z)**



# LTI DT SYSTEM

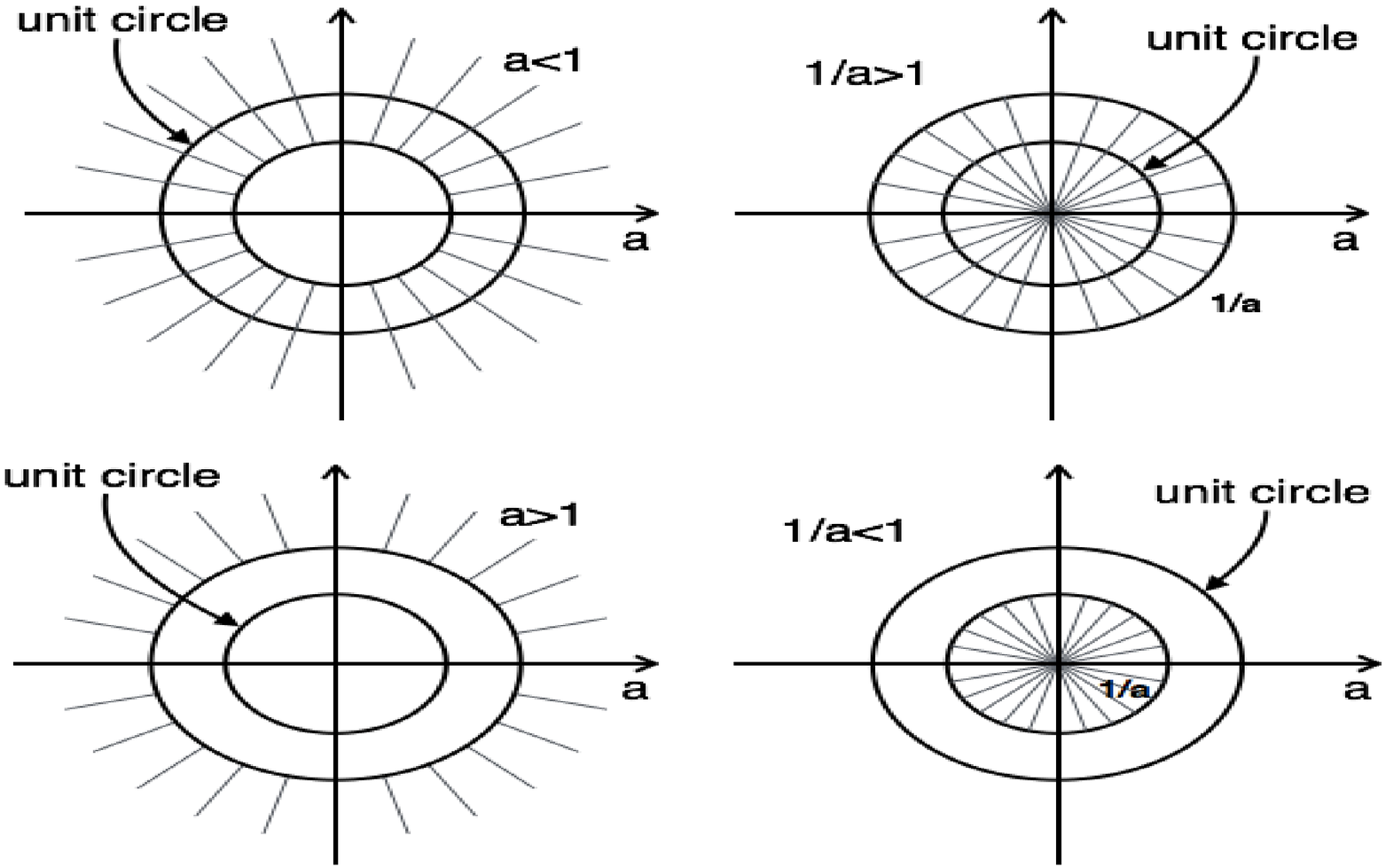
- **System Transfer Function:** Ratio of the output to the input.

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

- **Frequency Response:**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

# Z TRANSFORM – UNIT CIRCLE ROC



- **Solving Difference Equation using Z transform**

Shifting Property of Unilateral Z Transform:

$$y(n-1) \leftrightarrow Z^{-1} Y(Z) + Z\, y(-1)$$

$$y(n-2) \leftrightarrow Z^{-2} Y(Z) + Z^{-1} y(-1) + Z\, y(-2)$$

$$y(n-3) \leftrightarrow Z^{-3} Y(Z) + Z^{-2} y(-1) + Z^{-1} y(-2) + Z\, y(-3)$$

# Z TRANSFORM

Determine z transform of $x(n) = u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$= \sum_{n=0}^{\infty} 1 \, z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \cdots z^{-\infty}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots$$

$$= \left(1 - \frac{1}{z}\right)^{-1} \rightarrow \left(\frac{z-1}{z}\right)^{-1}$$

$$X(z) = \frac{z}{z-1} \; ; \; |z| > 1$$

$x(n) = \delta(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$X(z) = 1$$

# APPLICATIONS OF Z TRANSFORM

- It is used to analysis of discrete time systems.
- It is used for the digital signals
- It can be used to solve difference equations with constant coefficients
- To characterize the transfer function of discrete time LTI systems
- To design digital filter

# ASSESSMENT

1. What is meant by DTFT?
2. What is meant by linear convolution?
3. y(n) = x (n) * h(n) = h(n) * x(n) is defined as ---------- property
4. Define Z transform.
5. ROC of Z Transform can be used to determine ---------- of the system.
6. The system transfer function of LTI DT system is ----------

# THANK YOU