SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with ‘A++’ Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

23ECT202 – SIGNALS AND SYSTEMS

II YEAR/ III SEMESTER

UNIT 5 – LTI DISCRETE TIME SYSTEMS

TOPIC – LTI SYSTEM ANALYSIS USING Z TRANSFORM

DIFFERENCE EQUATION

- **Difference Equation:** It is an efficient way to implement discrete time systems
- The convolution of input sequence x(n) and unit sample response h(n) gives the output y(n)

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

- Two types of systems depending upon the length of unit sample response h(k)

- **Finite Impulse Response (FIR) Systems:** Unit sample response (or) Impulse response h(n) has finite no. of terms

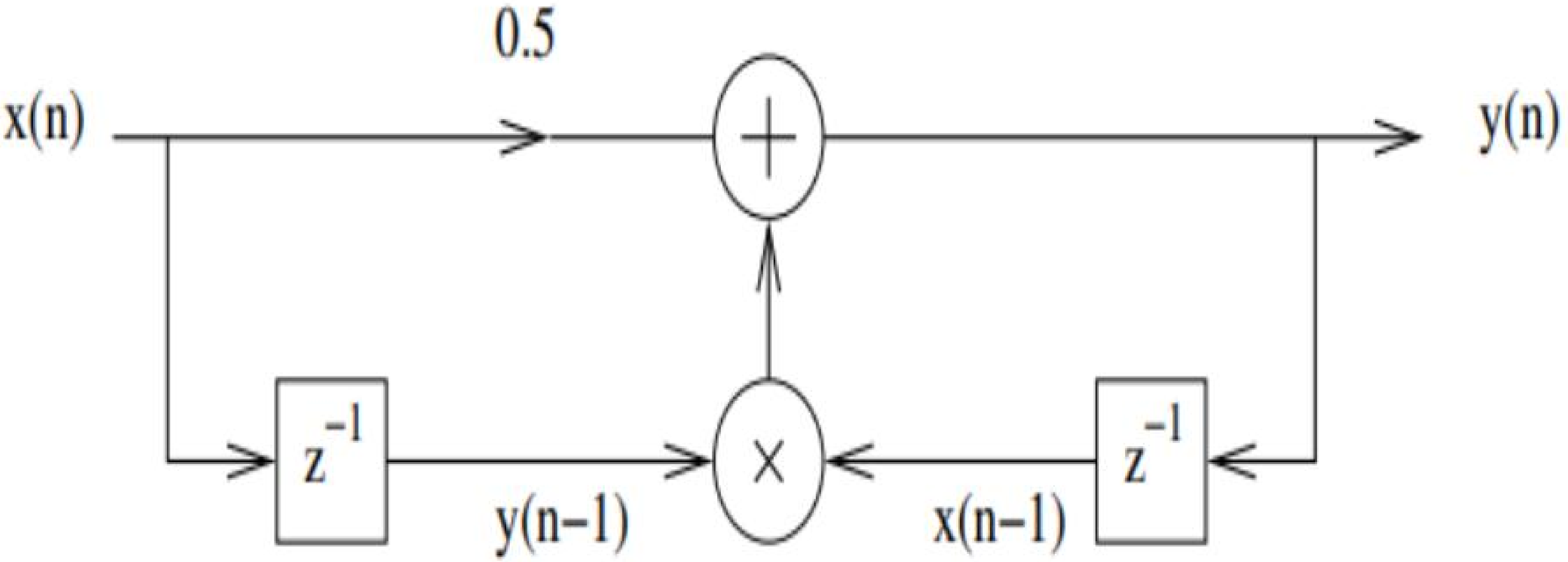
$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

- **Infinite Impulse Response (IIR) Systems:** Length of Unit sample response (or) Impulse response h(n) is infinite

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

RECURSIVE DIFFERENCE EQUATION

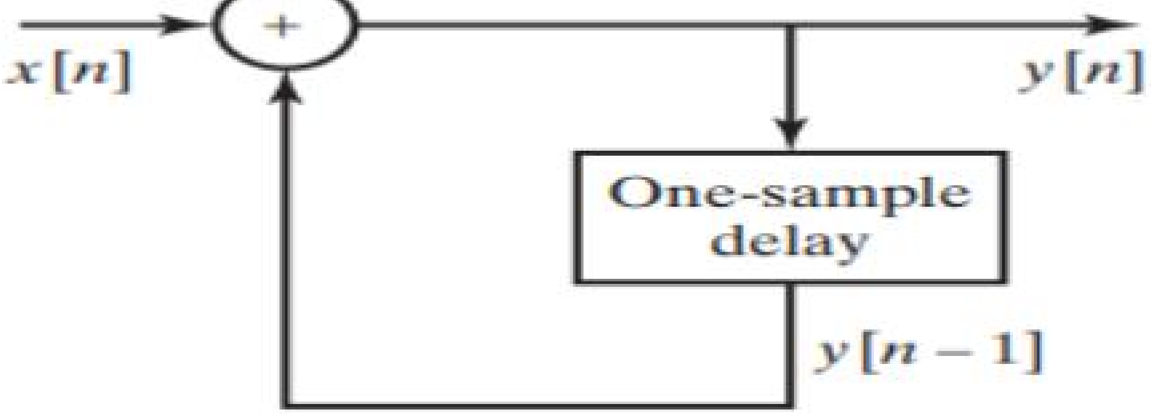
$$y(n) = y(n-1)\,x(n-1) + 0.5\,x(n)$$



- **Recursive Systems:** Output y(n) depends on present and past inputs as well as past output

$$y(n) = \sum_{k=0}^{n} x(k)$$

- **Non Recursive Systems:** Output y(n) depends on present and past input.

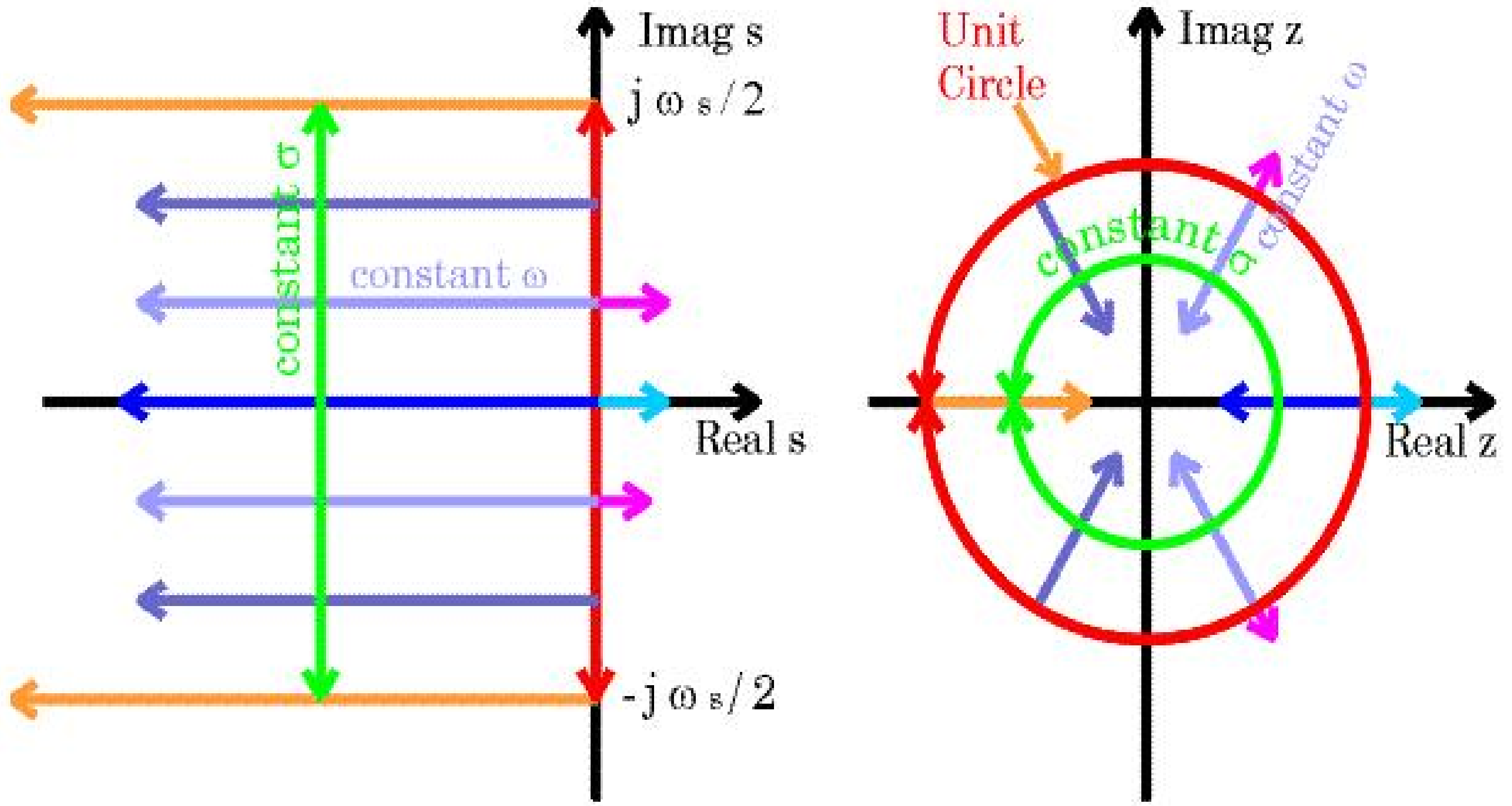


$$y(n) = \sum_{k=0}^{M} h(k)\, x(n-k)$$

Z TRANSFORM

- Z transform is used for the analysis of discrete time signals.
- It is more broad compared to Discrete Time Fourier Transform
- It is very much useful in discrete time signals as well as system analysis
- x(n) and X(Z) is called Z transform pair

x(n) ⟺ X(Z)



- **System Transfer Function:** Ratio of the output to the input.

$$H(Z) = \frac{Y(Z)}{X(Z)}$$

- **Frequency Response:**

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

LTI DISCRETE TIME SYSTEM

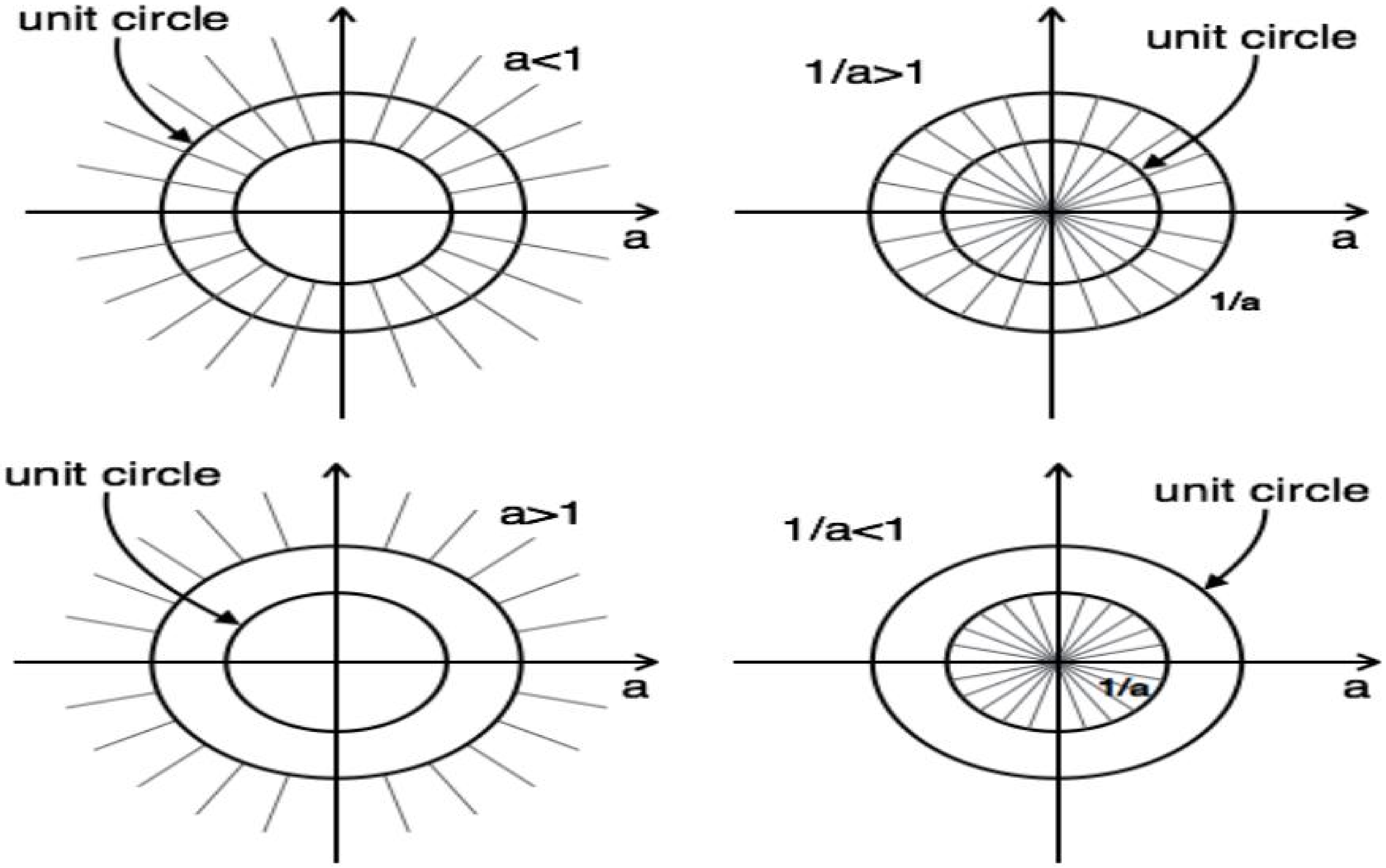
- Condition for an Linear Time Invariant (LTI) system to be causal:

$$\mathbf{h(n) = 0, n<0}$$

- Condition for an Linear Time Invariant (LTI) system to be stable:

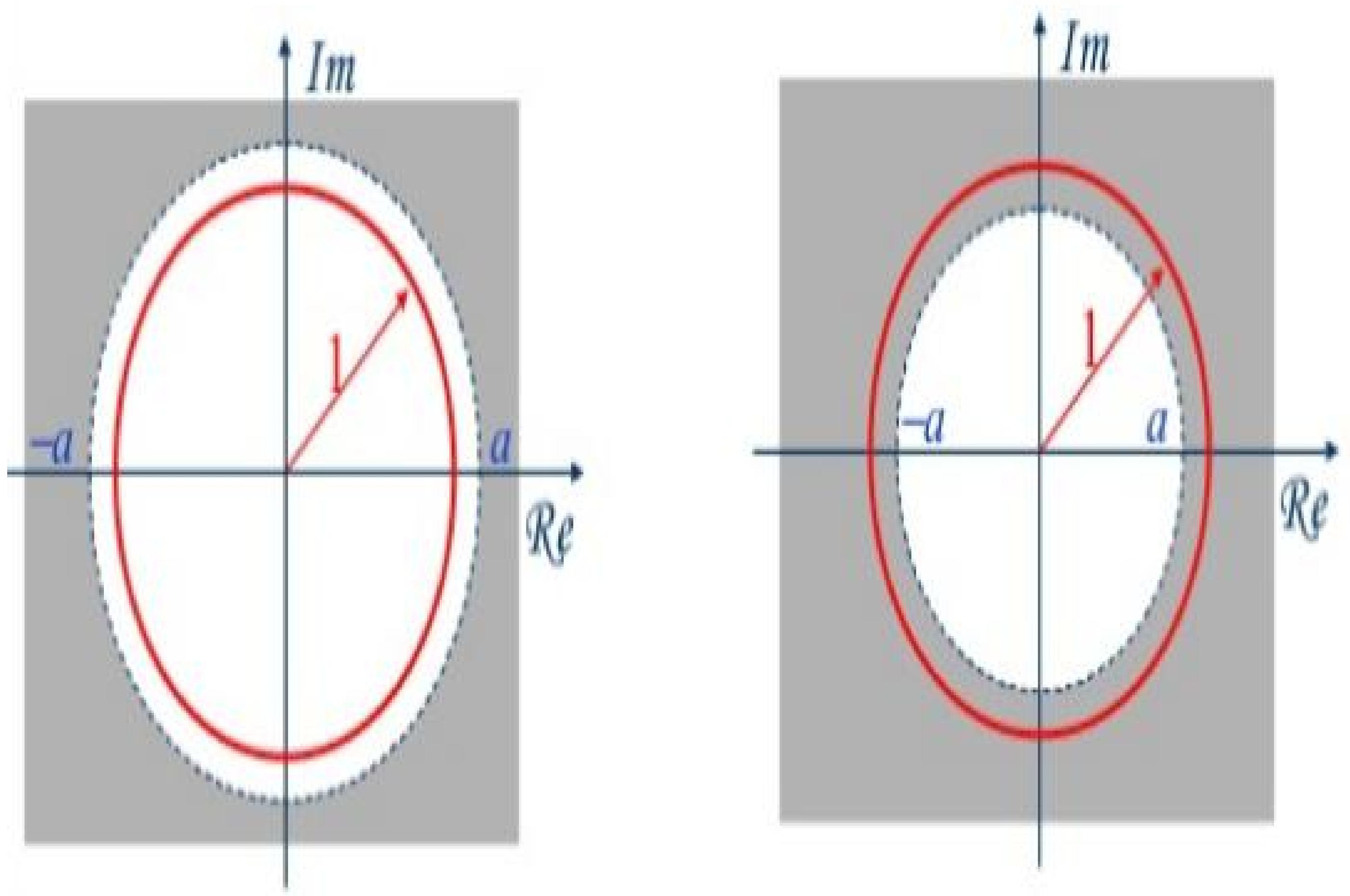
$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Z TRANSFORM – UNIT CIRCLE ROC

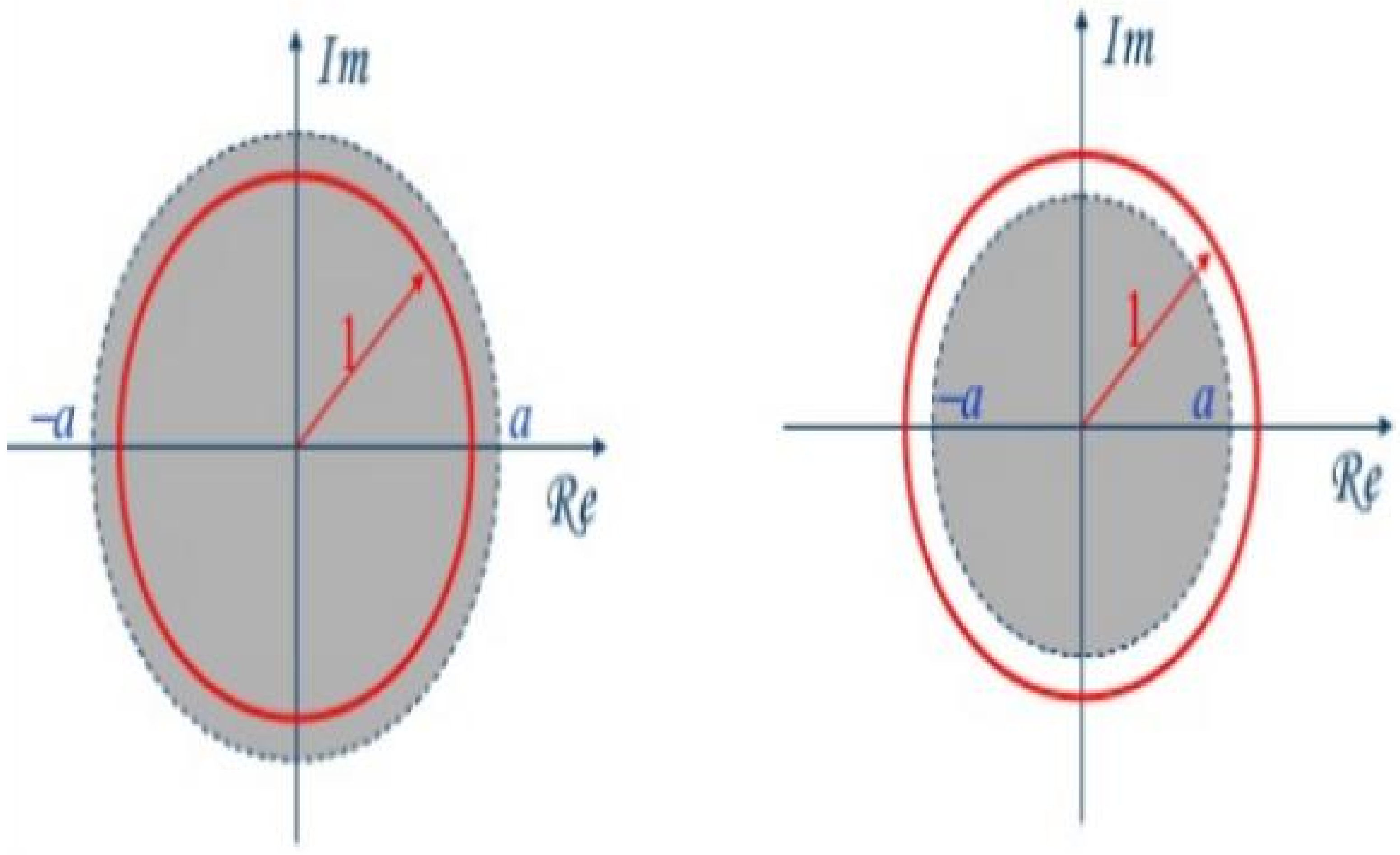


ROC OF Z TRANSFORM

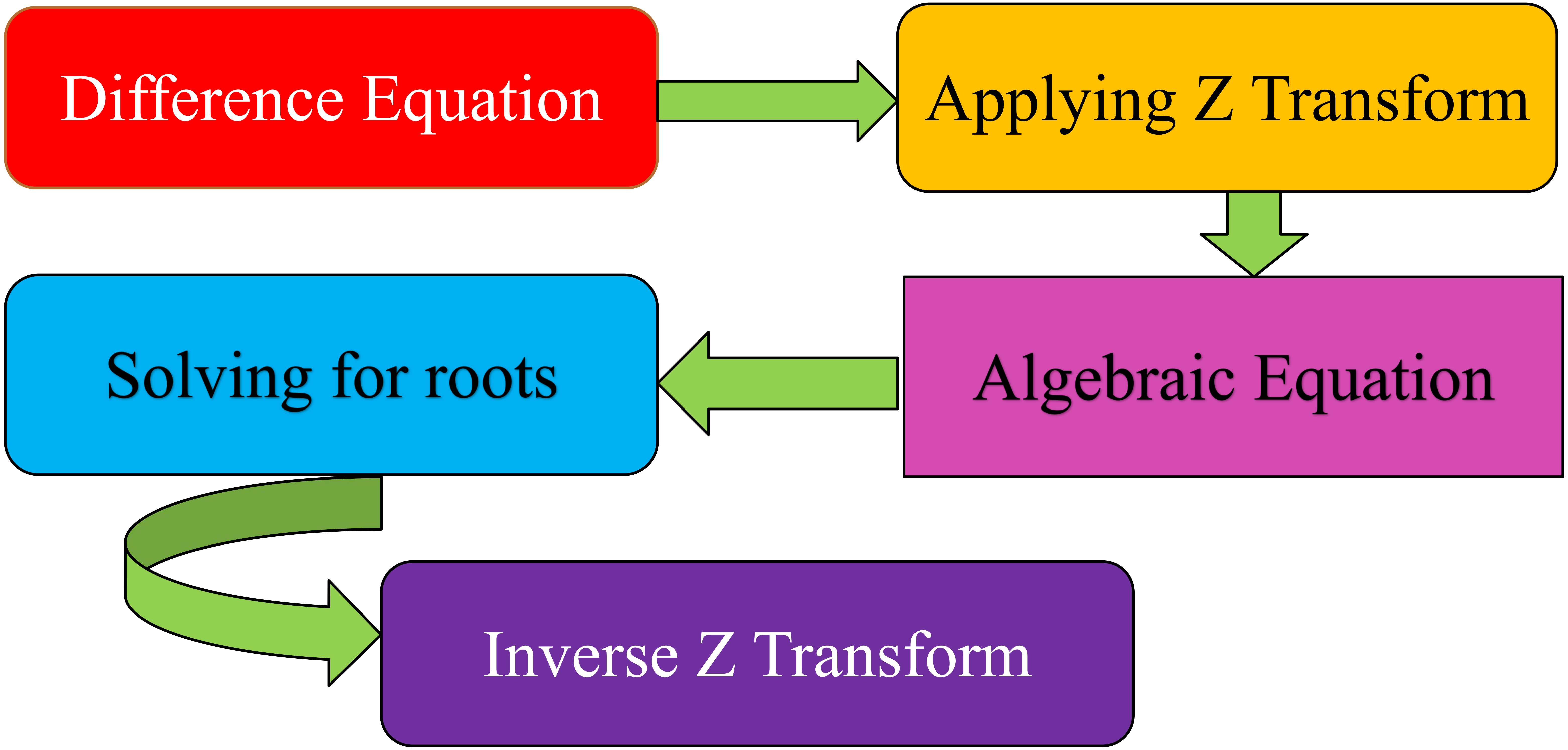
$$X(z)=\frac{z}{z-a}, \quad |z|>|a|$$



$$X(z)=\frac{z}{z-a}, \quad |z|<|a|$$

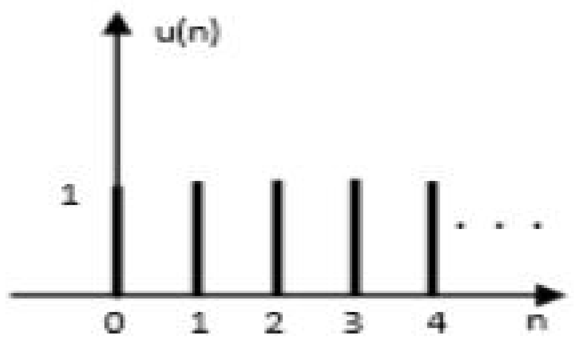


TO FIND IMPULSE RESPONSE

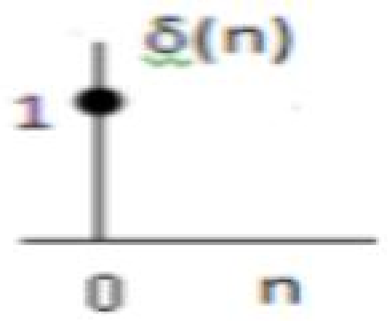


DISCRETE TIME SIGNALS

$$u(n) = 1 \text{ for } n \geq 0$$
$$= 0 \text{ for } n < 0$$



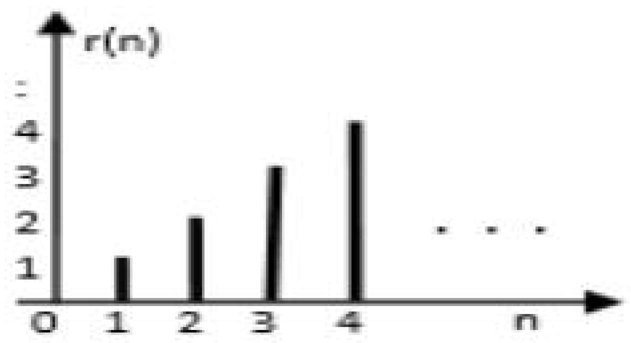
Unit step signal



Unit Impulse signal

$$\delta(n) = 1 \text{ for } n = 0$$
$$= 0 \text{ for } n \neq 0$$

$$r(n) = n \text{ for } n \geq 0$$
$$= 0 \text{ for } n < 0$$

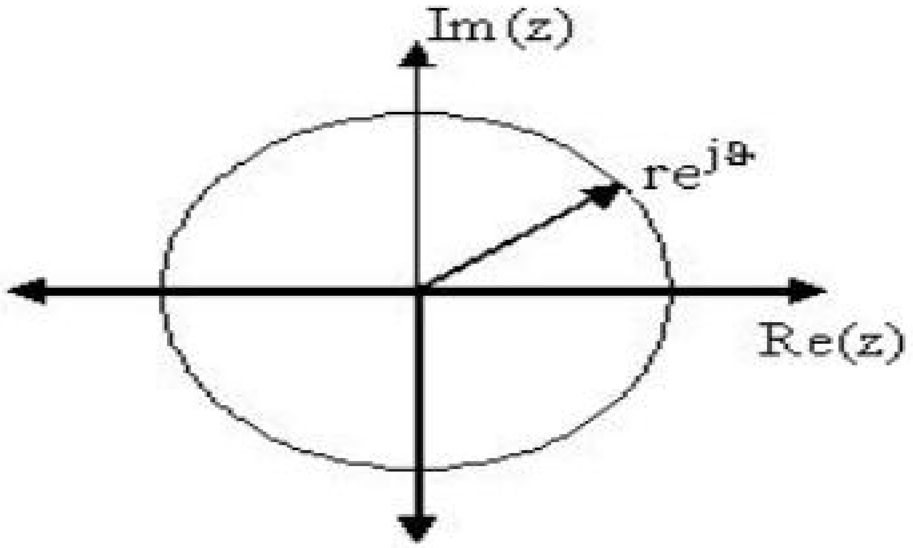


Unit Ramp signal

POLE ZERO PLOT OF LTI DT SYSTEM

- **Zeros:** The value(s) for z where P(z)=0.
- The complex frequencies that make the overall gain of the filter transfer function zero.
- **Poles:** The value(s) for z where Q(z)=0.
- The complex frequencies that make the overall gain of the filter transfer function infinite.

X(z)=P(z)/Q(z)



- **Solving Difference Equation using Z transform**

Shifting Property of Unilateral Z Transform:

$$y(n-1) \leftrightarrow Z^{-1} Y(Z) + Z\, y(-1)$$

$$y(n-2) \leftrightarrow Z^{-2} Y(Z) + Z^{-1} y(-1) + Z\, y(-2)$$

$$y(n-3) \leftrightarrow Z^{-3} Y(Z) + Z^{-2} y(-1) + Z^{-1} y(-2) + Z\, y(-3)$$

Z TRANSFORM

Determine z transform of $x(n) = u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$= \sum_{n=0}^{\infty} 1\, z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \dots z^{-\infty}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots$$

$$= \left(1 - \frac{1}{z}\right)^{-1} \rightarrow \left(\frac{z-1}{z}\right)^{-1}$$

$$X(z) = \frac{z}{z-1} \; ; \; |z| > 1$$

$x(n) = \delta(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$X(z) = 1$$

LTI DISCRETE TIME SYSTEM

$y(n) = 2\left(\frac{1}{3}\right)^n u(n)$ & $x(n) = u(n)$ Find Impulse Response

$Y(z) = 2\left(\frac{z}{z-\frac{1}{3}}\right)$ $X(z) = \frac{z}{z-1}$ $\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{2(z-1)}{z-\frac{1}{3}}$

$\frac{H(z)}{z} = \frac{2(z-1)}{z\left(z-\frac{1}{3}\right)} \Rightarrow \frac{A}{z} + \frac{B}{z-\frac{1}{3}}$

$A = 6$ $B = -4$

$H(z) = 6\left(\frac{z}{z}\right) - 4\left(\frac{z}{z-\frac{1}{3}}\right)$

$\therefore h(n) = 6\,\delta(n) - 4\left(\frac{1}{3}\right)^n u(n)$

LTI DT SYSTEM TRANSFER FUNCTION

$y(n) = 7y(n-1) - 12y(n-2) + 2x(n) - x(n-2)$ – System Transfer function

$Y(z) = 7z^{-1}Y(z) - 12z^{-2}Y(z) + 2X(z) - z^{-2}X(z)$

$Y(z) - 7z^{-1}Y(z) + 12z^{-2}Y(z) = X(z)\left[2 - z^{-2}\right]$

$Y(z)\left[1 - 7z^{-1} + 12z^{-2}\right] = X(z)\left[2 - z^{-2}\right]$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-2}}{1 - 7z^{-1} + 12z^{-2}}$$

APPLICATIONS OF Z TRANSFORM

- It is used to analysis of discrete time systems.
- It is used for the digital signals
- It can be used to solve difference equations with constant coefficients
- To characterize the transfer function of discrete time LTI systems
- To design digital filter

ASSESSMENT

1. Define Difference Equation.
2. Impulse response h(n) has finite no. of terms is called --------------
3. What is meant by recursive systems.
4. Ratio of the output to the input is called -------------
5. List the steps to find the impulse response of LTI DT System.
6. Z transform of unit step signal is ------------
7. List the applications of Z transform.
8. Name the condition for an Linear Time Invariant (LTI) system to be causal.

THANK YOU