



DEPARTMENT OF MATHEMATICS

UNIT III

Envelope: A curve which touches each member of a family of curves is called the envelope of that family curves.

Problems:

(1) Find the envelope of the following :

(i) $y = mx + \frac{1}{m}$ (ii) $y = mx + \sqrt{a^2m^2 + b^2}$

(iii) $y = mx + \frac{3}{2m}$ (iv) $(x-d)^2 + (y-d)^2 = 2d$

Solution:

(i) $y = mx + \frac{1}{m}$

$$y = \frac{m^2x + 1}{m}$$

$$my = m^2x + 1$$

$m^2x - my + 1 = 0$, which is quadratic in 'm'.

$A = x, B = -y, C = 1$

Envelope: $B^2 - 4AC = 0 \Rightarrow (-y)^2 - 4(x)(1) = 0$
 $y^2 - 4x = 0$

$$\boxed{y^2 = 4x}$$

(ii) $y = mx + \sqrt{a^2m^2 + b^2}$

$$y - mx = \sqrt{a^2m^2 + b^2}$$

Squaring, $(y - mx)^2 = a^2m^2 + b^2$



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$$y^2 + m^2 x^2 - 2ymx = a^2 m^2 + b^2$$

$$y^2 + m^2 x^2 - 2ymx - a^2 m^2 - b^2 = 0$$

$$m^2(x^2 - a^2) - 2ymx + y^2 - b^2 = 0$$

Here $A = x^2 - a^2$, $B = -2ymx$, $C = y^2 - b^2$

Envelope: $B^2 - 4AC = 0$

$$(-2ymx)^2 - 4(x^2 - a^2)(y^2 - b^2) = 0$$

$$4y^2x^2 - 4[x^2y^2 - x^2b^2 - a^2y^2 + a^2b^2] = 0$$

$$y^2x^2 - x^2y^2 + x^2b^2 + a^2y^2 - a^2b^2 = 0$$

$$x^2b^2 + a^2y^2 = a^2b^2$$

\div by $a^2b^2 \Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

(iii) $y = mx + \frac{3}{2m}$

$$y = \frac{2m^2x + 3}{2m}$$

$$2my = 2m^2x + 3$$

$$2m^2x - 2my + 3 = 0$$

$A = 2x$, $B = -2y$, $C = 3$

$$B^2 - 4AC = 0$$

$$\Rightarrow 4y^2 - 4(2x)(3) = 0$$

$$y^2 - 6x = 0$$

$$\boxed{y^2 = 6x}$$

(iv) $(x - \alpha)^2 + (y - \alpha)^2 = 2\alpha$

$$x^2 + \alpha^2 - 2\alpha x + y^2 - 2\alpha y + \alpha^2 = 2\alpha$$

$$2\alpha^2 - 2\alpha(x + y + 1) + (x^2 + y^2) = 0$$

$\Rightarrow A = 2$, $B = -2(x + y + 1)$
 $C = x^2 + y^2$

$$B^2 - 4AC = 0$$

$$\Rightarrow 4(x + y + 1)^2 - 4(2)(x^2 + y^2) = 0$$

$$\Rightarrow \boxed{(x + y + 1)^2 = 2(x^2 + y^2)}$$



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Envelope of two parameters:

① Find the envelope of the family of straight lines

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ where } a \text{ and } b \text{ are connected by the}$$

relation (i) $a+b=c$ (ii) $ab=c^2$ where c is a constant.

Solution:

$$(i) \quad \frac{x}{a} + \frac{y}{b} = 1 \rightarrow (1)$$

$$a+b=c \rightarrow (2)$$

$$\Rightarrow b=c-a$$

subs 'b' in (1),

$$\frac{x}{a} + \frac{y}{c-a} = 1$$

$$x(c-a) + ya = a(c-a)$$

$$xc - ax + ya = ac - a^2$$

$$xc - ax + ya - ac + a^2 = 0$$

$$a^2 + a(y-x-c) + cx = 0.$$

Here $A=1$, $B=y-x-c$, $C=cx$

Envelope: $B^2 - 4AC = 0$

$$(y-x-c)^2 - 4(1)(cx) = 0$$

$$(y-x-c)^2 - 4cx = 0$$

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$$(ii) \quad \frac{x}{a} + \frac{y}{b} = 1 \rightarrow (1)$$

$$ab = c^2 \rightarrow (2)$$

$$\Rightarrow b = c^2/a$$

Subs 'b' in (1),

$$\frac{x}{a} + \frac{y}{c^2/a} = 1$$

$$\frac{x}{a} + \frac{ay}{c^2} = 1$$

$$c^2x + a^2y = ac^2$$

$$a^2y - ac^2 + c^2x = 0$$

$$A = y, B = -c^2, c = c^2x$$

$$B^2 - 4AC = 0$$

$$(c^2)^2 - 4(y)(c^2x) = 0$$

$$c^4 - 4xy c^2 = 0$$

$$\div \text{ by } c^2 \Rightarrow c^2 - 4xy = 0$$

$$\boxed{4xy = c^2}$$