



DEPARTMENT OF MATHEMATICS

UNIT IV

Method of Lagrangian Multipliers

Working rule :

* Let $f(x, y, z)$ be the function of three variables subject to the constraint $\phi(x, y, z) = 0$

* Set the auxiliary function as,

$$g = f + \lambda \phi$$

where λ is the Lagrangian multiplier.

* Set $\frac{\partial g}{\partial x} = 0$, $\frac{\partial g}{\partial y} = 0$, $\frac{\partial g}{\partial z} = 0$

* On solving the above equations, we get the values of x, y and z . Substituting x, y & z in ϕ we get value of λ . They are called stationary values.

* On substitution of x, y and z in the given function we get the required extreme values.

Problems

① Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$

Soln:



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp;
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & amp; B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

$$f(x, y, z) = x^2 + y^2 + z^2$$

$$\phi(x, y, z) = x + y + z - 3a$$

Let the auxiliary function be,

$$g = f + \lambda \phi$$

$$g = x^2 + y^2 + z^2 + \lambda(x + y + z - 3a)$$

$$\frac{\partial g}{\partial x} = 0 \Rightarrow 2x + \lambda = 0 \Rightarrow -2x = -\lambda \rightarrow \textcircled{1}$$

$$\frac{\partial g}{\partial y} = 0 \Rightarrow 2y + \lambda = 0 \Rightarrow 2y = -\lambda \rightarrow \textcircled{2}$$

$$\frac{\partial g}{\partial z} = 0 \Rightarrow 2z + \lambda = 0 \Rightarrow 2z = -\lambda \rightarrow \textcircled{3}$$

From $\textcircled{1}$, $\textcircled{2}$ & $\textcircled{3}$,

$$2x = 2y = 2z$$

$$\Rightarrow x = y = z \rightarrow \textcircled{4}$$

$$\text{Given: } x + y + z = 3a$$

$$x + x + x = 3a \quad (\text{using } \textcircled{4})$$

$$3x = 3a$$

$$\boxed{x = a}$$

$$\therefore \boxed{x = y = z = a}$$

(a, a, a) is the point where minimum value occurs.

$$\therefore \text{Minimum Value} = x^2 + y^2 + z^2 = a^2 + a^2 + a^2 \\ = 3a^2$$