



DEPARTMENT OF MATHEMATICS

UNIT V

Double integration in Cartesian Coordinates :

① Evaluate $\int_0^1 \int_1^2 x(x+y) dx dy$

$$= \int_0^1 \left[\frac{x^3}{3} + y \cdot \frac{x^2}{2} \right]_1^2 dy$$

$$= \int_0^1 \left(\frac{8}{3} - \frac{1}{3} + \frac{y}{2} (4-1) \right) dy$$

$$= \int_0^1 \left(\frac{7}{3} + 2y - \frac{y}{2} \right) dy = \left[\frac{7}{3}y + 2 \frac{y^2}{2} - \frac{1}{2} \frac{y^2}{2} \right]_0^1$$

$$= \frac{7}{3} + 1 - \frac{1}{4} = \frac{37}{12}$$

② Evaluate $\int_0^3 \int_0^2 e^{x+y} dy dx$

$$= \int_0^3 \left[e^{x+y} \right]_0^2 dx = \int_0^3 (e^{x+2} - e^{x+0}) dx$$

$$= \left[e^{x+2} - e^x \right]_0^3 = e^5 - e^3 - e^2 + 1$$

③ Evaluate $\int_2^b \int_2^a \frac{dx dy}{xy}$

$$= \int_2^b \frac{dy}{y} (\log x)_2^a = (\log a - \log 2) \int_2^b \frac{dy}{y}$$

$$= (\log a - \log 2) (\log y)_2^b$$

$$= \log \left(\frac{a}{2} \right) \log \left(\frac{b}{2} \right)$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

(4) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dy dx$

$$= \int_0^a [y]_0^{\sqrt{a^2-x^2}} dx = \int_0^a \sqrt{a^2-x^2} dx$$

$$= \left[\frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]_0^a$$

$$= 0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - 0$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{4}$$

(5) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy dx}{1+x^2+y^2}$ $\left[\because \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right]$

$$= \int_0^1 \left[\int_0^{\sqrt{1+x^2}} \frac{dy}{1+x^2+y^2} \right] dx$$

$$= \int_0^1 \left[\int_0^{\sqrt{1+x^2}} \frac{dy}{(\sqrt{1+x^2})^2+y^2} \right] dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} \tan^{-1} \left(\frac{y}{\sqrt{1+x^2}} \right) \right]_0^{\sqrt{1+x^2}} dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}} [\tan^{-1}(1) - \tan^{-1}(0)] \right] dx$$

$$= \frac{\pi}{4} \int_0^1 \frac{dx}{\sqrt{1+x^2}} = \frac{\pi}{4} \left[\log \left(\frac{x+\sqrt{1+x^2}}{2+\sqrt{5}} \right) (x+\sqrt{1+x^2}) \right]$$

$$= \frac{\pi}{4} [\log(1+\sqrt{2})]$$