UNIT V

Change of order of integration

(1) Change the order of integration for $\int_0^1 \int_0^x f(x,y)\,dx\,dy$

Solution :

Given integral is not in the correct order.

Let us rearrange it.

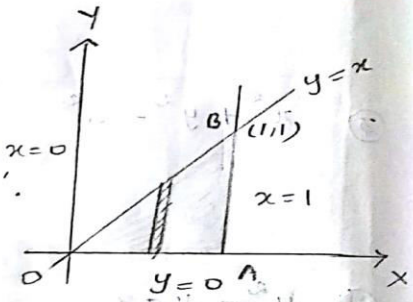
$$I = \int_0^1 \int_0^x f(x,y)\,dy\,dx$$

Given : $y = 0$ to $y = x$

$x = 0$ to $x = 1$

Inner limit is w.r.t y.

∴ It is a vertical strip

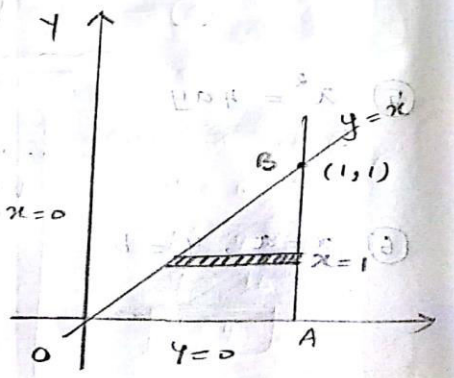


Now to change the order of integration we have to draw a horizontal strip.

x limits : $x = y$ to $x = 1$

y limits : $y = 0$ to $y = 1$

$$\therefore I = \int_0^1 \int_y^1 f(x,y)\,dx\,dy$$



(2) Change the order of integration in

$$\int_0^1 \int_0^y f(x,y)\,dx\,dy$$

Soln:

$$I = \int_0^1 \int_0^y f(x,y)\,dx\,dy$$

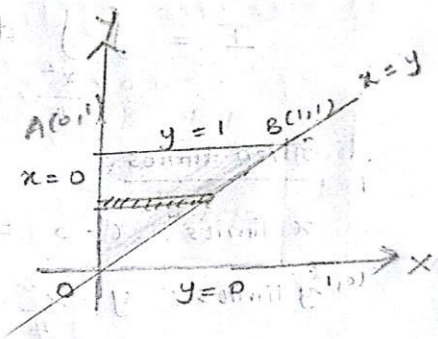
Given limits:

x limit : $x = 0$ to $x = y$

y limit : $y = 0$ to $y = 1$

Inner limit is w.r.t 'x'.

∴ Given limit is a horizontal strip.

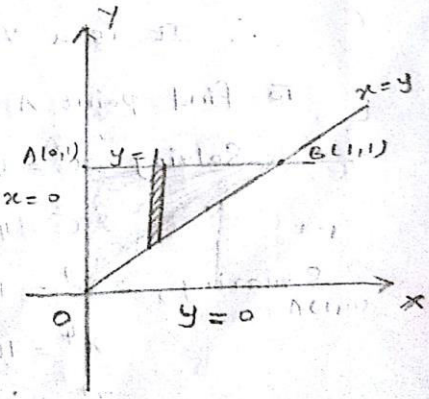


Now to change the order of integration we have to draw a vertical strip.

x limits : $x = 0$ to $x = 1$

y limits : $y = x$ to $y = 1$

$$\therefore I = \int_0^1 \int_x^1 f(x,y)\,dy\,dx$$



(3) Evaluate by changing the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$

Soln:

$$I = \int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\, dx$$

Given limits:

x limits: $x = 0$ to $x = 4$

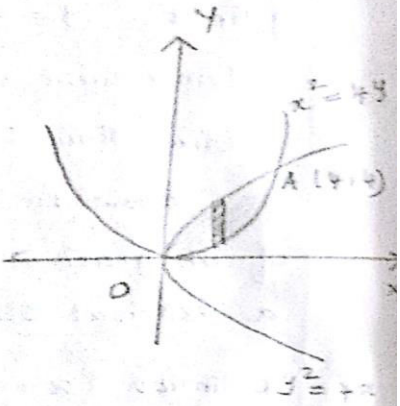
y limits: $y = \frac{x^2}{4}$ to $y = 2\sqrt{x}$

$4y = x^2$ to $y^2 = 4x$

i.e., $x^2 = 4y$ to $y^2 = 4x$

Inner limit is w.r.t 'y'

∴ It is a vertical strip



To find point A:

Solving $x^2 = 4y$ → (1) and $y^2 = 4x$ → (2)

$x^2 = 4y$

Squaring, $x^4 = 16y^2$

$x^4 = 16(4x)$ (using (2))

$x^4 = 64x$

$x^3 = 64$

$\boxed{x = 4}$

Subs x in (2),

$y^2 = 4x$

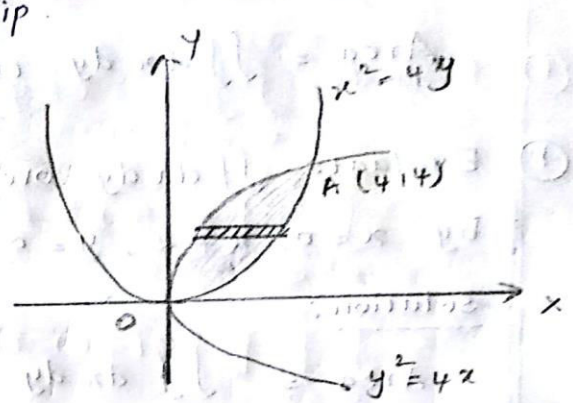
$y^2 = 4 \times 4$

$y^2 = 16$

$\boxed{y = 4}$

A (4, 4)

Now to change the order of integration draw a horizontal strip



x limits :

$x_1 = \frac{y^2}{4}$ to $x = 2\sqrt{y}$

$y = 0$ to $y = 4$

$$\therefore I = \int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} dx\, dy$$

$$= \int_0^4 \left[x \right]_{\frac{y^2}{4}}^{2\sqrt{y}} dy$$

$$= \int_0^4 \left[2\sqrt{y} - \frac{y^2}{4} \right] dy$$

$$= \left[2\,\frac{y^{3/2}}{3/2} - \frac{1}{4} \cdot \frac{y^3}{3} \right]_0^4$$

$$= \frac{4}{3}\left(4^{3/2}\right) - \frac{1}{12}\left(4^3\right)$$

$$= \frac{4}{3} \times 8 - \frac{1}{12/3}(4 \times 4 \times 4)$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$\boxed{I = \frac{16}{3}}$$

$4^{3/2} = 4\sqrt{4}$
$= 4 \times 2$
$= 8$