

SIS

(An Autonomous Institution)

DEPARTMENT OF MATHEMATICS

UNIT IV

JACOBIANS

If u = f(x,y), v = g(x,y) are two functions then Jacobian of u and v w.r.t x,y is denoted by $\frac{\partial(u,v)}{\partial(x,y)}$ (or) J and is defined by,

$$J = \frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

For three variables (u, v, w) which are functions of (x, y, z) then Jacobian is,

$$J = \frac{\partial (u, v, w)}{\partial (x, y, z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$



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PROPERTIES OF JACOBIANS:
* If u and v are functions of x and y and
I and y are functions of rand s, then
$\partial(u,v)$ $\partial(u,v)$ $\partial(x,y)$
$\frac{\partial(u,v)}{\partial(r,s)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,s)}$
* u and v are functions of x and y, then
$\partial(u,v)$. $\partial(x,y) = 1 \Rightarrow JJ'=1$
∂(x,y) ∂(u,v)
* Functional dependence
If $\partial(u,v,w) = 0$, then u,v and w
then Jacopius of a and (z,y,x) & z,y is desifted
are functionally dependent.
PROBLEMS: . (Ex)6
1) Find the Jacobian of the following transformation
(i) $u = 2x - y$, $v = y/2$. Find $\frac{\partial(u, v)}{\partial v}$
2 cariables (u, v, u) and are
(ii) $u = xyz$, $v = xy+yz+zx$, $w = x+y+z$.
Find D(U, V, W)
$\partial(x,y,z)$
(iii) $u = 2\pi y$, $v = x^2/y$ \ Find $\partial(u, v)$
(iv) $u = y^2/x$, $v = x^2/y$ $\partial(x,y)$
(V) $x = r \cos \theta$, $y = r \sin \theta$, $z = z$ Find $\frac{\partial (x_1 y_1 z)}{\partial (r_1 \theta_1 z)}$
(vi) x = u(1+v), y = v(1+u). Find \(\partial(\alpha, y)\)
$\partial(u,v)$





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(vii)
$$x = \frac{u^2}{v}$$
, $y = \frac{v^2}{w}$, $z = \frac{w^2}{u}$. Find $\frac{\partial}{\partial}(x,y,z)$ $\frac{\partial}{\partial}(u,v,w)$
(viii) $u = e^x \cos y$, $v = e^x \sin y$. Find $\frac{\partial}{\partial}(u,v,w)$ $\frac{\partial}{\partial}(x,y)$
(ix) $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$. Find $\frac{\partial}{\partial}(u,v,w)$ $\frac{\partial}{\partial}(x,y,z)$
(i) $J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$
(ii) $J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix} = \frac{\partial}{\partial}(u,v,w)$
 $= \begin{vmatrix} yz & xz & xy \\ y+z & x+z & x+y \\ 1 & 1 & 1 \end{vmatrix}$
 $= (x-y)(y-z)(z-x)$
(iii) $J = \begin{vmatrix} \frac{\partial}{\partial}(u,v) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) \\ \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) \\ \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) \\ \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y) \\ \frac{\partial}{\partial}(x,y) & \frac{\partial}{\partial}(x,y)$



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$$(iv) \ J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} -y^2/x^2 & 2y/x \\ 2x/y & -x^2/y^2 \end{vmatrix} = -3$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{J} = -\frac{1}{5}$$

$$(v) \ J = \frac{\partial(x,y,z)}{\partial(x,0,z)} = \begin{vmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial x} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} \cos \theta & -r\sin \theta & 0 \\ \sin \theta & r\cos \theta & 0 \end{vmatrix}$$

$$= r$$

$$(vii) \ J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1+v & u \\ v & 1+u \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{1+v + v}{v^2}$$

$$= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \frac{2u}{v^2} - \frac{u^2}{v^2} = 0$$

$$= \frac{2v}{u^2} - \frac{v^2}{u^2} = \frac{v^2}{u^2} = \frac{2v}{u^2}$$

$$= \frac{2v}{u^2} - \frac{v^2}{u^2} = \frac{2v}{u^2} = \frac{2v}{u^2}$$

(Viii)
$$J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} e^{x}\cos y & -e^{x}\sin y \\ e^{x}\sin y & e^{x}\cos y \end{vmatrix}$$

(ix) $J = \begin{vmatrix} -yz & z & y \\ x^{2} & x & \frac{y}{x} \end{vmatrix}$

$$= e^{2x}$$

$$= \frac{2x}{x^{2}}$$

$$= \frac{2x}$$

If $u = 2\pi y$, $v = x^2 - y^2$ and $x = r \cos \theta$, $y = r \sin \theta$ Evaluate $\frac{\partial(r, \theta)}{\partial(u, v)}$.

Evaluate
$$\frac{\partial(r,0)}{\partial(u,v)}$$

Soln:

$$\frac{\partial(u,v)}{\partial(r,0)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,0)}$$

$$= \left| \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y} \right| \left| \frac{\partial x}{\partial r} \cdot \frac{\partial x}{\partial 0} \right|$$

$$\frac{\partial v}{\partial x} \cdot \frac{\partial v}{\partial y} \left| \frac{\partial v}{\partial r} \cdot \frac{\partial y}{\partial 0} \right|$$

$$= \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} \begin{vmatrix} \cos 0 & -r\sin 0 \\ \sin 0 & r\cos 0 \end{vmatrix}$$

$$= -48^3$$

$$\frac{\partial(r_10)}{\partial(u_1v)} = \frac{1}{\frac{\partial(u_1v)}{\partial(r_10)}} = \frac{-1}{4r^3}$$

3 If
$$x+y+z=u$$
, $y+z=uv$, $z=uvw$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.