



DEPARTMENT OF MATHEMATICS

UNIT IV

MAXIMA AND MINIMA

WORKING RULE TO FIND MAXIMUM OR MINIMUM VALUES [EXTREMUM VALUES] OF $f(x, y)$.

* Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

* Set $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

Solve it Simultaneously

The solution point of these equations are called stationary points.

* Find the values of $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$,

$t = \frac{\partial^2 f}{\partial y^2}$ at these points.



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- * (a) If $rt - s^2 > 0$ and $r < 0$, then the function is maximum at that point.
- (b) If $rt - s^2 > 0$ and $r > 0$, then the function is minimum at that point.
- (c) If $rt - s^2 < 0$, then the function is neither maximum nor minimum at that point. This point is called as saddle point.
- (d) If $rt - s^2 = 0$, then the case is inconclusive. Hence further investigation is required.

NECESSARY CONDITION:

The necessary condition for the function $f(x, y)$ to have a maxima minima at a point (a, b) is $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$ at (a, b) .

SUFFICIENT CONDITION:

Write 3 and 4 step in working rule. This is the sufficient condition for the function to be maxima or minima.



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PROBLEMS :

① Examine $f(x, y) = x^3 + y^3 - 12x - 3y + 20$ for its extreme values.

Soln:

$$f(x, y) = x^3 + y^3 - 12x - 3y + 20$$

$$\frac{\partial f}{\partial x} = 3x^2 - 12 \quad ; \quad \frac{\partial f}{\partial y} = 3y^2 - 3$$

$$\frac{\partial f}{\partial x} = 0 \quad \text{and} \quad \frac{\partial f}{\partial y} = 0$$

$$3x^2 - 12 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$3y^2 - 3 = 0$$

$$y^2 = 1$$

$$y = \pm 1$$

Hence the stationary points are $(2, 1)$, $(2, -1)$, $(-2, 1)$, $(-2, -1)$.

$$r = \frac{\partial^2 f}{\partial x^2} = 6x$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$t = \frac{\partial^2 f}{\partial y^2} = 6y$$

Critical Points	$r = 6x$	$s = 0$	$t = 6y$	$rt - s^2$	Conclusion
$(2, 1)$	$12 > 0$	0	6	$72 > 0$	Minimum point
$(2, -1)$	$12 > 0$	0	-6	$-72 < 0$	Neither max nor min point
$(-2, 1)$	$-12 < 0$	0	6	$-72 < 0$	- Saddle point
$(-2, -1)$	$-12 < 0$	0	-6	$72 > 0$	Maximum point



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$$\text{Min value} = [f(x, y)]_{(2,1)} = 0$$

$$\text{Max value} = [f(x, y)]_{(-2,-1)} = 38$$

② Find the maximum and minimum values of $x^2 - xy + y^2 - 2x + y$.

Soln:

$$\frac{\partial f}{\partial x} = 0, \quad \frac{\partial f}{\partial y} = 0$$

$$2x - y - 2 = 0, \quad -x + 2y + 1 = 0$$

$$\Rightarrow x = 1, \quad y = 0$$

$(1, 0)$ is the stationary point.

$$r = \frac{\partial^2 f}{\partial x^2} = 2 > 0$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = -1 < 0$$

$$t = \frac{\partial^2 f}{\partial y^2} = 2 > 0$$

$$rt - s^2 = 4 - 1 = 3 > 0$$

$(1, 0)$ is a minimum point.

$$\text{Min. value} = [f(x, y)]_{(1,0)} = -1.$$