



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



DEPARTMENT OF MATHEMATICS

UNIT IV

Taylor's expansion

Let $f(x, y)$ be a function of two variables x and y .

The Taylor Series expansion about the point (a, b) is given by,

$$\begin{aligned}
f(x, y) = & f(a, b) + (x-a) f_x(a, b) + (y-b) f_y(a, b) \\
& + \frac{1}{2!} \left[(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b) f_{xy}(a, b) + (y-b)^2 f_{yy}(a, b) \right] \\
& + \frac{1}{3!} \left[(x-a)^3 f_{xxx}(a, b) + 3(x-a)^2(y-b) f_{xxy}(a, b) \right. \\
& \left. + 3(x-a)(y-b)^2 f_{xyy}(a, b) + (y-b)^3 f_{yyy}(a, b) \right] \\
& + \dots
\end{aligned}$$

Problems :

- ① Expand $x^2y + 3y - 2$ in powers of $x-1$ and $y+2$ using Taylor's expansion.

Solution :

$$\text{Given : } f(x, y) = x^2y + 3y - 2$$

$$\therefore a = 1$$

$$b = -2$$

$$x-1 = 0$$

$$x = 1$$

$$y+2 = 0$$

$$y = -2$$



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$f(x, y) = x^2y + 3y - 2$	At $(a, b) = (1, -2)$
$f(x, y) = x^2y + 3y - 2$	$f(1, -2) = 1(-2) + 3(-2) - 2$ $= -10$
$f_x = 2xy$	$f_x(1, -2) = 2(1)(-2) = -4$
$f_{xx} = 2y$	$f_{xx}(1, -2) = 2(-2) = -4$
$f_{xxx} = 0$	$f_{xxx}(1, -2) = 0$
$f_y = x^2 + 3$	$f_y(1, -2) = 1 + 3 = 4$
$f_{yy} = 0$	$f_{yy}(1, -2) = 0$
$f_{yyy} = 0$	$f_{yyy}(1, -2) = 0$
$f_{xy} = 2x$	$f_{xy}(1, -2) = 2(1) = 2$
$f_{xxy} = 2$	$f_{xxy}(1, -2) = 2$
$f_{xyy} = 0$	$f_{xyy}(1, -2) = 0$
Taylor's Series expansion is,	
$f(x, y) = f(a, b) + (x-a)f_x(a, b) + (y-b)f_y(a, b)$ $+ \frac{1}{2!} [(x-a)^2 f_{xx}(a, b) + 2(x-a)(y-b)f_{xy}(a, b) + (y-b)^2 f_{yy}(a, b)]$ $+ \frac{1}{3!} [(x-a)^3 f_{xxx}(a, b) + 3(x-a)^2(y-b)f_{xxy}(a, b)$ $+ 3(x-a)(y-b)^2 f_{xyy}(a, b) + (y-b)^3 f_{yyy}(a, b)]$ $+ \dots$	
$f(x, y) = f(1, -2) + (x-1)f_x(1, -2) + (y+2)f_y(1, -2)$ $+ \frac{1}{2!} [(x-1)^2 f_{xx} + 2(x-1)(y+2)f_{xy} + (y+2)^2 f_{yy}]$	



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$$+ \frac{1}{3!} \left[(x-1)^3 f_{xxx} + 3(x-1)^2(y+2) f_{xxy} + \right. \\ \left. 3(x-1)(y+2)^2 f_{xyy} + (y+2)^3 f_{yyy} \right] + \dots$$

$$\therefore f(x,y) = -10 + (x-1)(-4) + (y+2)4 + \\ \frac{1}{2} \left[(x-1)^2(-4) + 2(x-1)(y+2)2 + 0 \right] \\ + \frac{1}{6} \left[0 + 3(x-1)^2(y+2)2 + 0 + 0 \right]$$

$$f(x,y) = -10 - 4(x-1) + 4(y+2) + \\ \frac{1}{2} \left[-4(x-1)^2 + 4(x-1)(y+2) \right] \\ + \frac{1}{6} \left[6(x-1)^2(y+2) \right] + \dots$$

$$f(x,y) = -10 - 4(x-1) + 4(y+2) - 2(x-1)^2 + \\ 2(x-1)(y+2) + (x-1)^2(y+2) + \dots$$