SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT V

① Double Integration in Cartesian Coordinates:

① Evaluate $\int_0^2 \int_0^2 x(x+y)\, dx\, dy$

$= \int_0^2 \left[\frac{x^3}{3} + y \cdot \frac{x^2}{2} \right]_1^2 dy$

$= \int_0^2 \left(\frac{8}{3} - \frac{1}{3} + \frac{y}{2}(4-1) \right) dy$

$= \int_0^2 \left(\frac{7}{3} + 2y - \frac{y}{2} \right) dy = \left[\frac{7}{3} y + 2\frac{y^2}{2} - \frac{1}{2} \frac{y^2}{2} \right]_0^1$

$= \frac{7}{3} + 1 - \frac{1}{4} = 37/12$

② Evaluate $\int_0^3 \int_0^2 e^{x+y}\, dy\, dx$

$= \int_0^3 \left[e^{x+y} \right]_0^2 dx = \int_0^3 (e^{x+2} - e^{x+0})\, dx$

$= \left[e^{x+2} - e^x \right]_0^3 = e^5 - e^3 - e^2 + 1$

③ Evaluate $\int_2^b \int_2^a \frac{dx\, dy}{xy}$

$= \int_2^b \frac{dy}{y} (\log x)_2^a = (\log a - \log 2) \int_2^b \frac{dy}{y}$

$= (\log a - \log 2)(\log y)_2^b$

$= \log\left(\frac{a}{2}\right) \log\left(\frac{b}{2}\right)$

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DEPARTMENT OF MATHEMATICS

(4) Evaluate $\int\limits_{0}^{a}\int\limits_{0}^{\sqrt{a^2-x^2}} dy\,dx$

$$= \int\limits_{0}^{a} [y]_{0}^{\sqrt{a^2-x^2}} dx = \int\limits_{0}^{a} \sqrt{a^2-x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{0}^{a}$$

$$= 0 + \frac{a^2}{2}\sin^{-1}(1) - 0 - 0$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{4}$$

(5) $\int\limits_{0}^{1}\int\limits_{0}^{\sqrt{1+x^2}} \dfrac{dy\,dx}{1+x^2+y^2}$ $\qquad \left[\because \int \dfrac{dx}{a^2+x^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)\right]$

$$= \int\limits_{0}^{1}\left[\int\limits_{0}^{\sqrt{1+x^2}} \frac{dy}{1+x^2+y^2}\right] dx$$

$$= \int\limits_{0}^{1}\left[\int\limits_{0}^{\sqrt{1+x^2}} \frac{dy}{\left(\sqrt{1+x^2}\right)^2+y^2}\right] dx$$

$$= \int\limits_{0}^{1}\left[\frac{1}{\sqrt{1+x^2}}\tan^{-1}\left(\frac{y}{\sqrt{1+x^2}}\right)\right]_{0}^{\sqrt{1+x^2}} dx$$

$$= \int\limits_{0}^{1}\left[\frac{1}{\sqrt{1+x^2}}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]\right] dx$$

$$= \frac{\pi}{4}\int\limits_{0}^{1}\frac{dx}{\sqrt{1+x^2}} = \frac{\pi}{4}\left[\log\left(x+\sqrt{1+x^2}\right)\right]_{0}^{1}$$

$$= \frac{\pi}{4}\left[\log(1+\sqrt{2})\right]$$