SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT V

Change of order of integration

(1) Change the order of integration for

$$\int_0^1 \int_0^x f(x,y)\, dx\, dy$$

Solution :

Given integral is not in the correct order.
Let us rearrange it.

$$I = \int_0^1 \int_0^x f(x,y)\, dy\, dx$$

Given : $y = 0$ to $y = x$
$\qquad\quad x = 0$ to $x = 1$

Inner limit is w.r.t y.

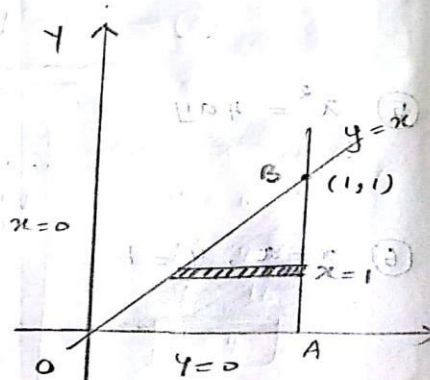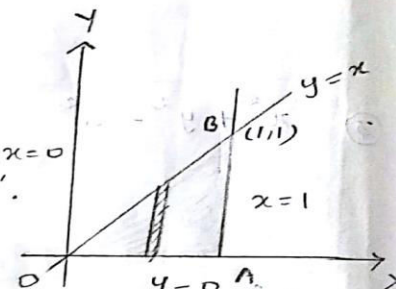∴ It is a vertical strip

Now to change the order
of integration we have to draw
a horizontal strip.

x limits : $x = y$ to $x = 1$

y limits : $y = 0$ to $y = 1$.

∴ $\boxed{I = \int_0^1 \int_y^1 f(x,y)\, dx\, dy}$

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② Change the order of integration in

$$\int_0^1 \int_0^y f(x,y)\, dx\, dy$$

Soln:

$$I = \int_0^1 \int_0^y f(x,y)\, dx\, dy$$

Given limits :

x limit : $x = 0$ to $x = y$

y limit : $y = 0$ to $y = 1$

Inner limit is w.r.t 'x'.

∴ Given limit is a horizontal strip.

Now to change the order of integration we have to draw a vertical strip.

x limits : $x = 0$ to $x = 1$

y limits : $y = x$ to $y = 1$

$$\therefore \quad I = \int_0^1 \int_x^1 f(x,y)\, dy\, dx$$

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③ Evaluate by changing the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\, dx$

Soln:

$$I = \int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\, dx$$

Given limits:

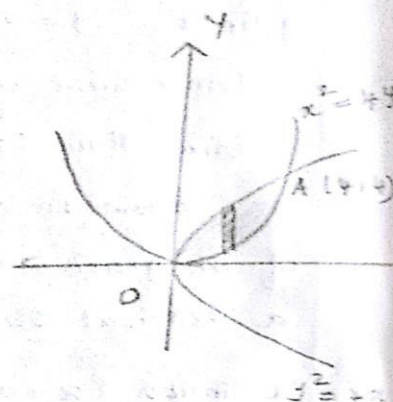x limits : $x = 0$ to $x = 4$

y limits : $y = \frac{x^2}{4}$ to $y = 2\sqrt{x}$

$$4y = x^2 \quad \text{to} \quad y^2 = 4x$$

i.e., $x^2 = 4y$ to $y^2 = 4x$

Inner limit is w.r.t 'y'

∴ It is a vertical strip

To find point A :

Solving $x^2 = 4y$ → ① and $y^2 = 4x$ → ②

$$x^2 = 4y$$

Squaring, $x^4 = 16 y^2$

$x^4 = 16 (4x)$ (using ②)

$x^4 = 64x$

$x^3 = 64$

$$\boxed{x = 4}$$

⇒ subs x in ②,

$y^2 = 4x$

$y^2 = 4 \times 4$

$y^2 = 16$

$$\boxed{y = 4}$$

A (4,4)

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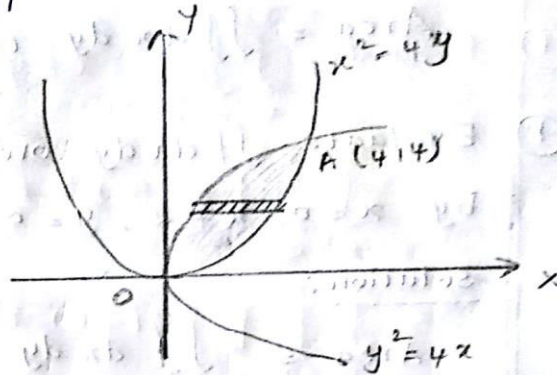*DEPARTMENT OF MATHEMATICS*

Now to change the order of integration draw a horizontal strip.

x limits :

$x = \dfrac{y^2}{4}$ to $x = 2\sqrt{y}$

$y = 0$ to $y = 4$

$$\therefore I = \int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} dx \, dy$$

$$= \int_0^4 \left[x \right]_{\frac{y^2}{4}}^{2\sqrt{y}} dy$$

$$= \int_0^4 \left[2\sqrt{y} - \frac{y^2}{4} \right] dy$$

$$= \left[2 \frac{y^{3/2}}{3/2} - \frac{1}{4} \cdot \frac{y^3}{3} \right]_0^4$$

$$= \frac{4}{3} \left(4^{3/2} \right) - \frac{1}{12} \left(4^3 \right)$$

$$= \frac{4}{3} \times 8 - \frac{1}{12} (4 \times 4 \times 4)$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$\boxed{I = \frac{16}{3}}$$

$4^{3/2} = 4\sqrt{4}$
$= 4 \times 2$
$= 8$