



## DEPARTMENT OF MATHEMATICS

### UNIT V

#### Change of order of integration

① Change the order of integration for

$$\int_0^1 \int_0^x f(x,y) dx dy$$

Solution:

Given integral is not in the correct order.

Let us rearrange it.

$$I = \int_0^1 \int_0^x f(x,y) dy dx$$

Given:  $y = 0$  to  $y = x$

$x = 0$  to  $x = 1$

Inner limit is w.r.t  $y$ .

∴ It is a vertical strip

Now to change the order

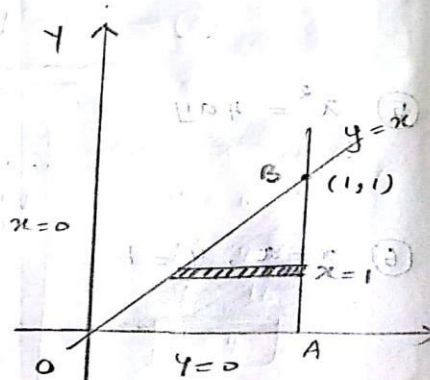
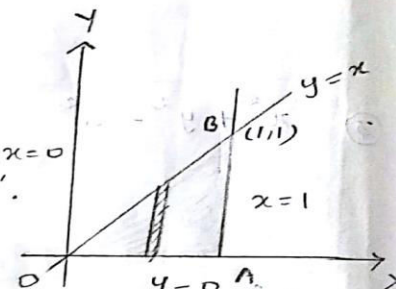
of integration we have to draw

a horizontal strip.

$x$  limits:  $x = y$  to  $x = 1$

$y$  limits:  $y = 0$  to  $y = 1$

$$\therefore I = \int_0^1 \int_y^1 f(x,y) dx dy$$





# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) & amp;

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & amp; B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



## DEPARTMENT OF MATHEMATICS

2 Change the order of integration in

$$\int_0^1 \int_0^y f(x,y) dx dy$$

Soln:

$$I = \int_0^1 \int_0^y f(x,y) dx dy$$

Given limits:

x limit :  $x = 0$  to  $x = y$

y limit :  $y = 0$  to  $y = 1$

Inner limit is w.r.t 'x'.

∴ Given limit is a horizontal strip.

Now to change the order of integration we have to draw a vertical strip.

x limits :  $x = 0$  to  $x = 1$

y limits :  $y = x$  to  $y = 1$

$$I = \int_0^1 \int_x^1 f(x,y) dy dx$$

The first diagram shows a Cartesian coordinate system with x and y axes. A line y=x passes through the origin. A horizontal line y=1 is drawn. The region bounded by x=0, y=0, and y=1 is shaded. A horizontal strip is drawn at a height y, extending from x=0 to x=y. Points A(0,1) and B(1,1) are marked on the line y=1. The origin is labeled O.

The second diagram shows the same Cartesian coordinate system. A vertical strip is drawn at a position x, extending from y=x to y=1. Points A(0,1) and B(1,1) are marked on the line y=1. The origin is labeled O.



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



## DEPARTMENT OF MATHEMATICS

③ Evaluate by changing the order of integration

$$\text{in } \int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy dx$$

Soln:

$$I = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy dx$$

Given limits:

x limits:  $x = 0$  to  $x = 4$

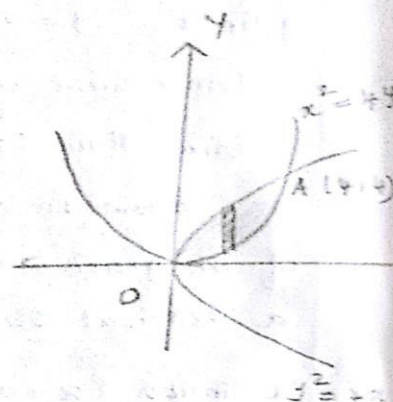
y limits:  $y = \frac{x^2}{4}$  to  $y = 2\sqrt{x}$

$4y = x^2$  to  $y^2 = 4x$

i.e.,  $x^2 = 4y$  to  $y^2 = 4x$

Inner limit is w.r.t 'y'

∴ It is a vertical strip



To find point A:

Solving  $x^2 = 4y$  and  $y^2 = 4x \rightarrow$  (2)

$x^2 = 4y$  (1)

Squaring,  $x^4 = 16y^2$

$x^4 = 16(4x)$  (using (2))

$x^4 = 64x$

$x^3 = 64$

$x = 4$

$A(4,4)$

∴ subs x in (2),

$y^2 = 4x$

$y^2 = 4 \times 4$

$y^2 = 16$

$y = 4$



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



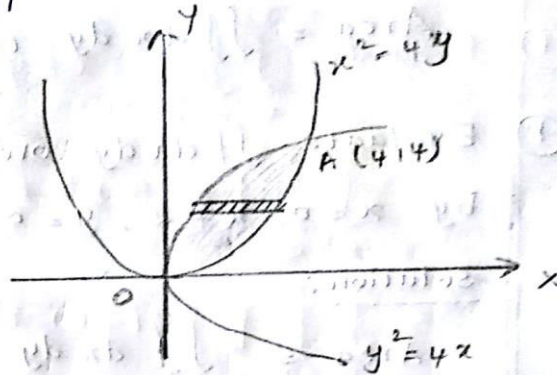
## DEPARTMENT OF MATHEMATICS

Now to change the order of integration draw a horizontal strip.

$x$  limits :

$$x = \frac{y^2}{4} \text{ to } x = 2\sqrt{y}$$

$$y = 0 \text{ to } y = 4$$



$$\therefore I = \int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} dx dy$$

$$= \int_0^4 \left[ x \right]_{\frac{y^2}{4}}^{2\sqrt{y}} dy$$

$$= \int_0^4 \left[ 2\sqrt{y} - \frac{y^2}{4} \right] dy$$

$$= \left[ 2 \frac{y^{3/2}}{3/2} - \frac{1}{4} \cdot \frac{y^3}{3} \right]_0^4$$

$$= \frac{4}{3} (4^{3/2}) - \frac{1}{12} (4^3)$$

$$= \frac{4}{3} \times 8 - \frac{1}{12} (4 \times 4 \times 4)$$

$$= \frac{32}{3} - \frac{16}{3}$$

$$\boxed{I = \frac{16}{3}}$$

$$\begin{aligned} 4^{3/2} &= 4\sqrt{4} \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$