



DEPARTMENT OF MATHEMATICS

UNIT V

Volume of triple integrals:

- ① Find the volume of the sphere:
 $x^2 + y^2 + z^2 = a^2$ without transformation.

Soln:

Volume = 8 x Volume in the 1st octant.

z limits:

$$x^2 + y^2 + z^2 = a^2$$

$$z^2 = a^2 - x^2 - y^2$$

$$z = \pm \sqrt{a^2 - x^2 - y^2}$$

z varies from 0 to $\sqrt{a^2 - x^2 - y^2}$

y limits:

Put $z = 0$ in $x^2 + y^2 + z^2 = a^2$

$$x^2 + y^2 = a^2$$

$$y^2 = a^2 - x^2$$

$$y = \pm \sqrt{a^2 - x^2}$$

y varies from 0 to $\sqrt{a^2 - x^2}$

x limit:

Put $y = 0, z = 0$ in $x^2 + y^2 + z^2 = a^2$

$$x^2 = a^2$$

$$x = \pm a$$

x varies from 0 to a.



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$$\begin{aligned}V &= 8 \int \int \int dz dy dx \\&= 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} dz dy dx \\&= 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} [z]_0^{\sqrt{a^2-x^2-y^2}} dy dx \\&= 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2} dy dx \\&= 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{(\sqrt{a^2-x^2})^2-y^2} dy dx \\&= 8 \int_0^a \left[\frac{y}{2} \sqrt{a^2-x^2-y^2} + \frac{a^2-x^2}{2} \sin^{-1} \left(\frac{y}{\sqrt{a^2-x^2}} \right) \right]_0^{\sqrt{a^2-x^2}} dx\end{aligned}$$

Formula :

$$\left[\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]$$

Here $a = \sqrt{a^2-x^2}$, $x = y$

$$\begin{aligned}&= 8 \int_0^a \left[0 + \frac{a^2-x^2}{2} \sin^{-1} \left(\frac{\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}} \right) - 0 \right] dx \\&= 8 \int_0^a \left(\frac{a^2-x^2}{2} \right) \sin^{-1}(1) dx \\&= 8 \int_0^a \left(\frac{a^2-x^2}{2} \right) \frac{\pi}{2} dx\end{aligned}$$



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$$V = \frac{8\pi}{2 \times 2} \int_0^a (a^2 - x^2) dx$$

$$= 2\pi \left[a^2 x - \frac{x^3}{3} \right]_0^a$$

$$= 2\pi \left[a^2 \cdot a - \frac{a^3}{3} \right]$$

$$= 2\pi \left[a^3 - \frac{a^3}{3} \right]$$

$$= 2\pi \left[\frac{3a^3 - a^3}{3} \right]$$

$$V = 2\pi \left(\frac{2a^3}{3} \right)$$

$$V = \frac{4\pi a^3}{3} \text{ cubic units}$$