



DEPARTMENT OF MATHEMATICS

UNIT IV

UNIT - IV
FUNCTIONS OF SEVERAL VARIABLES

PARTIAL DERIVATIVES

(1) Find the 1st order and 2nd order partial derivatives of $z = x^3 + y^3 - 3axy$.

Soln:

$$\frac{\partial z}{\partial x} = z_x = 3x^2 - 3ay$$

$$\frac{\partial z}{\partial y} = z_y = 3y^2 - 3ax$$

$$\frac{\partial^2 z}{\partial x^2} = z_{xx} = 6x$$

$$\frac{\partial^2 z}{\partial y^2} = z_{yy} = 6y$$

$$\frac{\partial^2 z}{\partial x \partial y} = z_{xy} = -3a$$

(2) Find the first order partial derivatives of $u = xe^y + ye^x$.

Soln:

$$\frac{\partial u}{\partial x} = e^y + ye^x$$

$$\frac{\partial u}{\partial y} = xe^y + e^x$$

(3) If $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$ then Show that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}$.



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Soln:

$$r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2 \rightarrow (1)$$

Diff (1) w.r.t x partially,

$$2r \frac{\partial r}{\partial x} = 2(x-a)$$

$$\Rightarrow \frac{\partial r}{\partial x} = \frac{x-a}{r}$$

$$\text{Similarly } \frac{\partial r}{\partial y} = \frac{y-b}{r}$$

$$\frac{\partial r}{\partial z} = \frac{z-c}{r}$$

$$\begin{aligned} \frac{\partial^2 r}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial r}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{x-a}{r} \right) \\ &= \frac{r(1) - (x-a) \frac{\partial r}{\partial x}}{r^2} \\ &= \frac{1}{r} - \frac{(x-a)}{r^2} \frac{\partial r}{\partial x} \end{aligned}$$

$$\therefore \frac{\partial^2 r}{\partial x^2} = \frac{1}{r} - \frac{(x-a)^2}{r^3} \rightarrow (2)$$

$$\text{Similarly } \frac{\partial^2 r}{\partial y^2} = \frac{1}{r} - \frac{(y-b)^2}{r^3} \rightarrow (3)$$

$$\frac{\partial^2 r}{\partial z^2} = \frac{1}{r} - \frac{(z-c)^2}{r^3} \rightarrow (4)$$

$$(2) + (3) + (4) \Rightarrow$$

$$\begin{aligned} \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} &= \frac{3}{r} - \frac{1}{r^3} [(x-a)^2 + (y-b)^2 + (z-c)^2] \\ &= \frac{3}{r} - \frac{1}{r^3} \cdot r^3 \\ &= \frac{2}{r} \end{aligned}$$



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④ If $z = (x^2 + xy + y^2)^r$ then Show that

$$\left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right) \left(x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}\right) = 4r^2 z.$$

Soln:

$$z = (x^2 + xy + y^2)^r$$

$$\frac{\partial z}{\partial x} = r(x^2 + xy + y^2)^{r-1} (2x + y)$$

$$\frac{\partial z}{\partial y} = r(x^2 + xy + y^2)^{r-1} (2y + x)$$

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = r(x^2 + xy + y^2)^{r-1} [2x^2 + 2y^2 + 2xy]$$

$$= 2r(x^2 + xy + y^2)^r$$

$$\begin{aligned} \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right) \left(x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}\right) &= \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}\right) (2r z) \\ &= (2r) (2r z) \\ &= 4r^2 z. \end{aligned}$$

⑤ If $z = f(x+ct) + g(x-ct)$ then prove that

$$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$

Soln:

$$\frac{\partial z}{\partial t} = f'(x+ct)(c) + g'(x-ct)(-c)$$

$$\frac{\partial^2 z}{\partial t^2} = c^2 [f''(x+ct) + g''(x-ct)]$$

$$\frac{\partial^2 z}{\partial x^2} = f''(x+ct) + g''(x-ct)$$

$$\therefore \frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$$