SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)
Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai
Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &
Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)
COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

UNIT V

Triple integration :

① Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z)\, dz\, dy\, dx$

$= \int_0^3 \int_0^2 \left[xz + yz + \frac{z^2}{2} \right]_0^1 dy\, dx$

$= \int_0^3 \int_0^2 \left[x(1-0) + y(1-0) + \frac{1}{2}(1-0) \right] dy\, dx$

$= \int_0^3 \int_0^2 \left[x + y + \frac{1}{2} \right] dy\, dx$

$= \int_0^3 \left[xy + \frac{y^2}{2} + \frac{1}{2} y \right]_0^2 dx$

$= \int_0^3 \left[x(2-0) + \frac{1}{2}(2^2-0) + \frac{1}{2}(2-0) \right] dx$

$= \int_0^3 \left[2x + 2 + 1 \right] dx$

$= \int_0^3 (2x + 3)\, dx$

$= \left[2 \cdot \frac{x^2}{2} + 3x \right]_0^3$

$= \left(3^2 - 0 + 3(3-0) \right)$
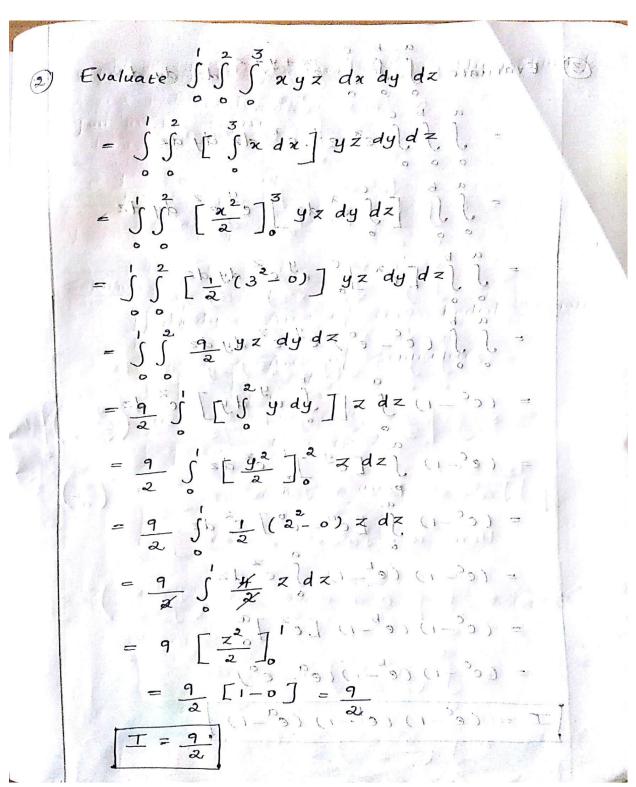
$= 9 + 9$

$\boxed{I = 18}$

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DEPARTMENT OF MATHEMATICS

(2) Evaluate $\int_0^1 \int_0^2 \int_0^3 xyz \, dx \, dy \, dz$

$$= \int_0^1 \int_0^2 \left[\int_0^3 x \, dx \right] yz \, dy \, dz$$

$$= \int_0^1 \int_0^2 \left[\frac{x^2}{2} \right]_0^3 yz \, dy \, dz$$

$$= \int_0^1 \int_0^2 \left[\frac{1}{2}(3^2 - 0) \right] yz \, dy \, dz$$

$$= \int_0^1 \int_0^2 \frac{9}{2} yz \, dy \, dz$$

$$= \frac{9}{2} \int_0^1 \left[\int_0^2 y \, dy \right] z \, dz$$

$$= \frac{9}{2} \int_0^1 \left[\frac{y^2}{2} \right]_0^2 z \, dz$$

$$= \frac{9}{2} \int_0^1 \frac{1}{2}(2^2 - 0) z \, dz$$

$$= \frac{9}{2} \int_0^1 \frac{4}{2} z \, dz$$

$$= 9 \left[\frac{z^2}{2} \right]_0^1$$

$$= \frac{9}{2} [1 - 0] = \frac{9}{2}$$

$$\boxed{I = \frac{9}{2}}$$

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③ Evaluate $\displaystyle\int_0^a \int_0^b \int_0^c e^{x+y+z} \, dx \, dy \, dz$

$$= \int_0^a \int_0^b \int_0^c e^x e^y \cdot e^z \, dx \, dy \, dz$$

$$= \int_0^a \int_0^b \left[\int_0^c e^x \, dx \right] e^y e^z \, dy \, dz$$

$$= \int_0^a \int_0^b [e^x]_0^c \, e^y e^z \, dy \, dz$$

$$= \int_0^a \int_0^b (e^c - e^0) \, e^y e^z \, dy \, dz$$

$$= (e^c - 1) \int_0^a \left[\int_0^b e^y \, dy \right] e^z \, dz$$

$$= (e^c - 1) \int_0^a [e^y]_0^b \, e^z \, dz$$

$$= (e^c - 1) \int_0^a [e^b - e^0] \, e^z \, dz$$

$$= (e^c - 1)(e^b - 1) \int_0^a e^z \, dz$$

$$= (e^c - 1)(e^b - 1) [e^z]_0^a$$

$$= (e^c - 1)(e^b - 1)(e^a - e^0)$$

$$\boxed{I = (e^c - 1)(e^b - 1)(e^a - 1)}$$