Maxima and Minima:

Necessary condition:

* The necessary condition for $f(x,y)$ to have maxima (or) minima at a point a, b is $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$

Sufficient condition:

* $A = \frac{\partial^2 f}{\partial x^2}$, $B = \frac{\partial^2 f}{\partial x \partial y}$, $C = \frac{\partial^2 f}{\partial y^2}$

* If $AC - B^2$ greather than 0 and $A>0$ the $f(x,y)$ has a minium value at $(a,b)$ $\boxed{\begin{matrix} AC-B^2 > 0 \\ A > 0 \end{matrix}}$ $AC-B^2 > 0$, $A < 0$

* If $AC - B^2 > 0$ and $A < 0$ then $f(x,y)$ has a maximum value at $(a,b)$

* If $AC - B^2 < 0$ then $f(x,y)$ has either maximum nor minimum value at $(a,b)$. This is called saddle point.

* If $AC - B^2 = 0$ then their is no conclusion. we need further investigation.

(*) very important

Critical point (or) stationary points

* A point A, B is called critical point (or) stationary point of $(x,y)$ if $\partial f/\partial x = 0$, $\partial f/\partial y = 0$.

Example: 1:

Find the maximum and minimum of $x^3+y^3-12x-3y+20$

Soln:

Given,

$$f(x,y) = x^3 + y^3 - 12x - 3y + 20.$$

$\frac{\partial f}{\partial x} = 0$

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 12/3$

$x^2 = 4$

$x = \pm 2$

$\frac{\partial f}{\partial y} = 0$

$3y^2 - 3 = 0$

$3y^2 = 3$

$y^2 = 3/3$

$y^2 = 1$

$y = \pm 1$

The critical points are $(2,1), (2,-1)$ $(-2,-1), (-2,1)$.

Necessary condition:

$A = \frac{\partial^2 f}{\partial x^2}$

$A = \frac{\partial^2 f}{\partial x^2}$

$= 6x$

$B = \frac{\partial^2 f}{\partial x \partial y}$

$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$

$= \frac{\partial}{\partial x} = (3y^2-3)$

$= 0$

$C = \frac{\partial^2 f}{\partial y^2}$

$C = \frac{\partial^2 f}{\partial y^2}$

$C = 6y$

| critical points | A | B | C | $AC - B^2$ | conclusion |
|---|---|---|---|---|---|
| (2,1) | $12>0$ | 0 | 6 | $72>0$ | Minimum |
| (2,-1) | $12>0$ | 0 | -6 | $-72<0$ | saddle point |
| (-2,1) | $-12<0$ | 0 | 6 | $-72<0$ | saddle point |
| (-2,-1) | $-12<0$ | 0 | -6 | $72>0$ | Maximum |

To find minimum (2,1)

$f(2,1) = 2^3 + 1^3 - 12(2) - 3(1) + 20$

$= 8 + 1 - 24 - 3 + 20$

$= 2$

To find maximum $(-2,-1)$.

$$f(-2,-1) = (-2)^3 + (-1)^3 - 12(-2) - 3(-1) + 20$$

$$= -8 - 1 + 24 + 3 + 20$$

$$= 38$$