



SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

COURSE NAME : 23ITT201 DATA STRUCTURES

II YEAR/ III SEMESTER

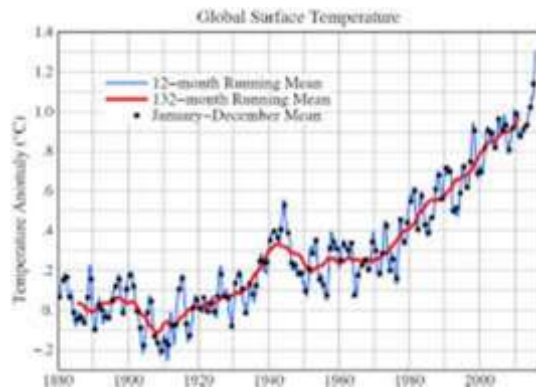
UNIT – IV MULTIWAY SEARCH TREES AND GRAPH

Topic: *GRAPH*

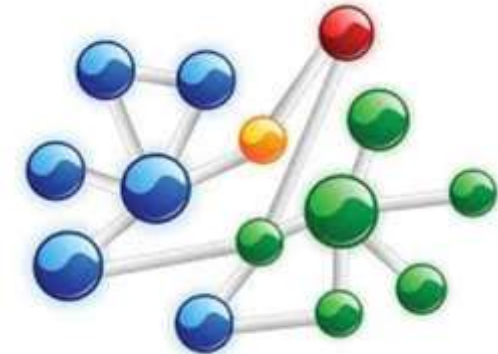


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UNIT IV





Introduction

- **Graph** is a collection of vertices and arcs which connects vertices in the graph
- Graph is a collection of nodes and edges which connects nodes in the graph
- Generally, a graph **G** is represented as **$G = (V , E)$** , where **V** is set of vertices and **E** is set of edges.



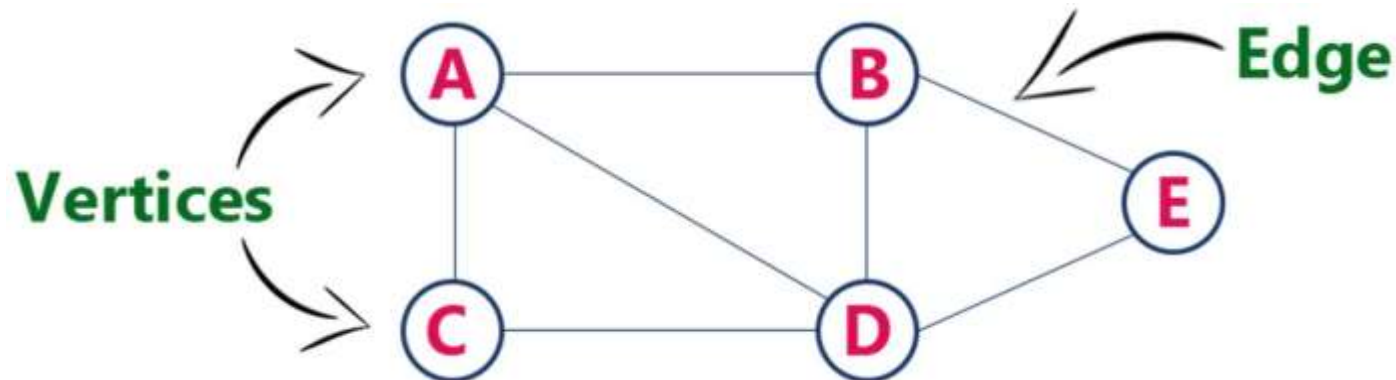
Introduction

The following is a graph with 5 vertices and 6 edges.

This graph G can be defined as $G = (V, E)$

Where $V = \{A, B, C, D, E\}$ and $E =$

$\{(A, B), (A, C), (A, D), (B, D), (C, D), (B, E), (E, D)\}$.





Basic Terminologies



Vertex

A individual data element of a graph is called as Vertex. **Vertex** is also known as **node**. In above example graph, A, B, C, D & E are known as vertices

Edge

An edge is a connecting link between two vertices. **Edge** is also known as **Arc**. An edge is represented as (starting Vertex, ending Vertex).

The link between vertices A and B is represented as (A,B). In above example graph, there are 7 edges (i.e., (A,B), (A,C), (A,D), (B,D), (B,E), (C,D), (D,E)).



Basic Terminologies



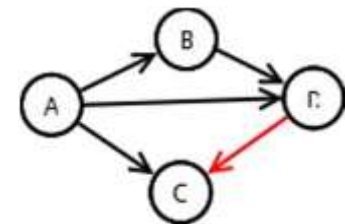
- **Undirected Edge** - An undirected edge is a bidirectional edge. If there is a undirected edge between vertices A and B then edge (A , B) is equal to edge (B , A).
- **Directed Edge** - A directed edge is a unidirectional edge. If there is a directed edge between vertices A and B then edge (A , B) is not equal to edge (B , A).
- **Weighted Edge** - A weighted edge is an edge with cost on it.

Undirected Graph

- A graph with only undirected edges is said to be undirected graph.

Directed Graph

- A graph with only directed edges is said to be directed graph.





Basic Terminologies



Mixed Graph

- A graph with undirected and directed edges is said to be mixed graph.

End vertices or Endpoints

- The two vertices joined by an edge are called the end vertices (or endpoints) of the edge.

Origin

- If an edge is directed, its first endpoint is said to be origin of it.

Destination

- If an edge is directed, its first endpoint is said to be origin of it and the other endpoint is said to be the destination of the edge.



Basic Terminologies



Adjacent

If there is an edge between vertices A and B then both A and B are said to be adjacent. In other words, Two vertices A and B are said to be adjacent if there is an edge whose end vertices are A and B.

Incident

An edge is said to be incident on a vertex if the vertex is one of the endpoints of that edge.

Outgoing Edge

A directed edge is said to be outgoing edge on its origin vertex.

Incoming Edge

A directed edge is said to be incoming edge on its destination vertex.



Basic Terminologies



➤ Degree of a Node

- In-degree: Number of edges pointing **to** a node
- Out-degree: Number of edges pointing **from** a node

➤ Path: sequence of vertices in which each pair of successive vertices is connected by an edge

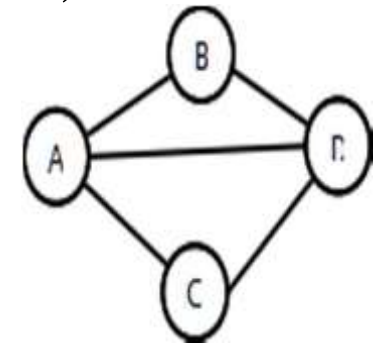
Cycle: a path that starts and ends on the same vertex

Simple path: a path that does not cross itself That is, no vertex is repeated (except first and last)

Simple paths cannot contain cycles

Length of a path: Number of edges in the

path Sometimes the sum of the weights of the edges

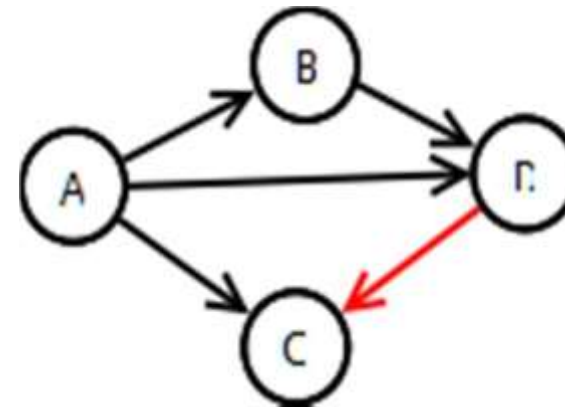
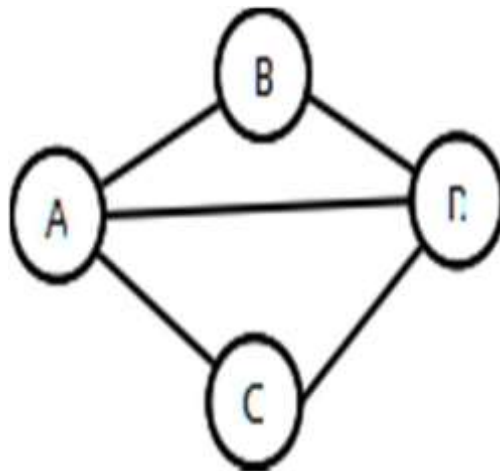




Basic Terminologies



- A **Cyclic** graph contains cycles Example: roads (normally)
An **acyclic** graph contains no cycles

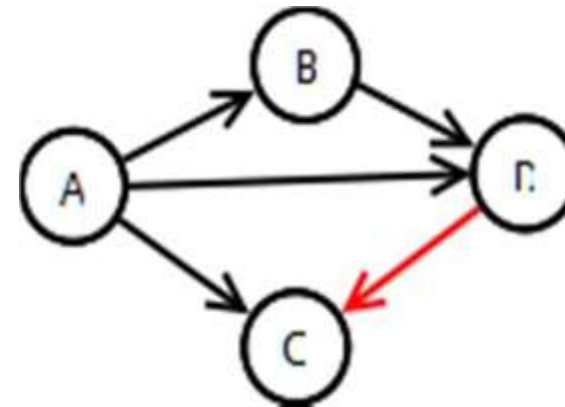
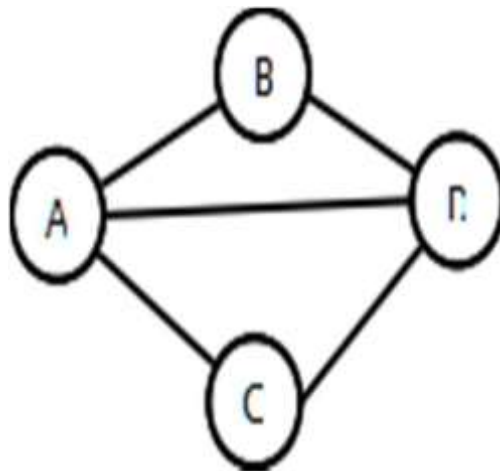




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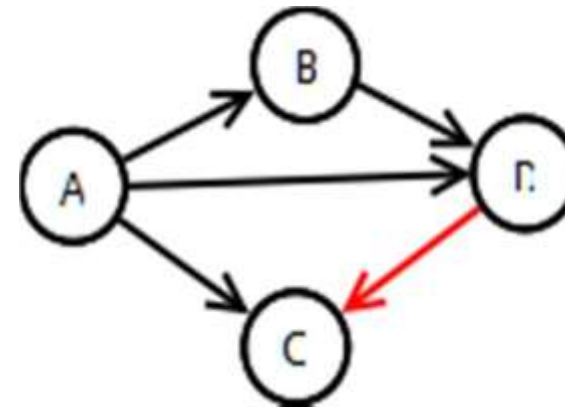
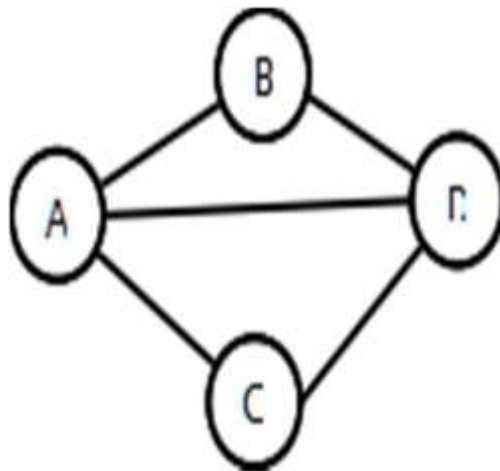




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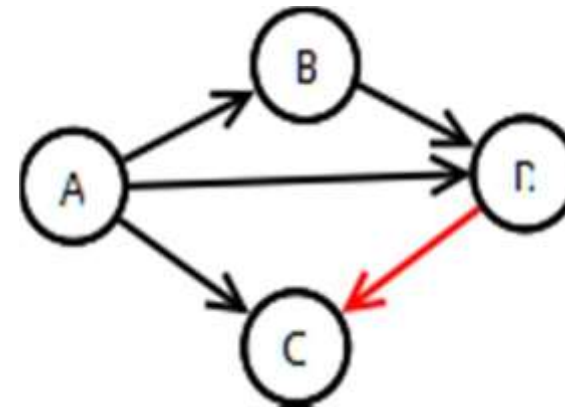
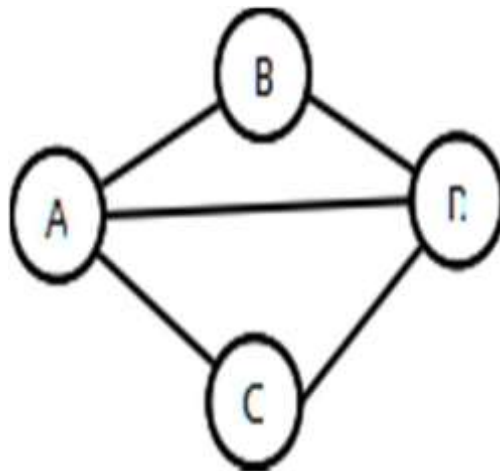




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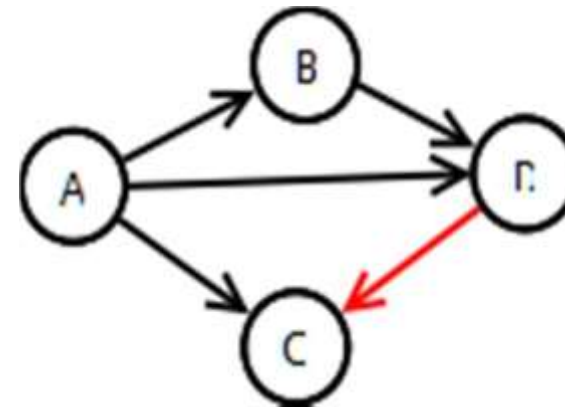
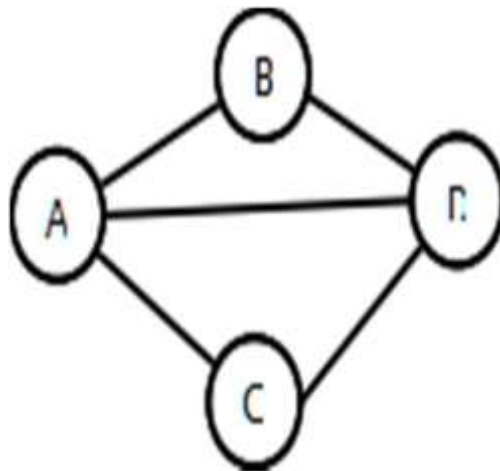




Basic Terminologies



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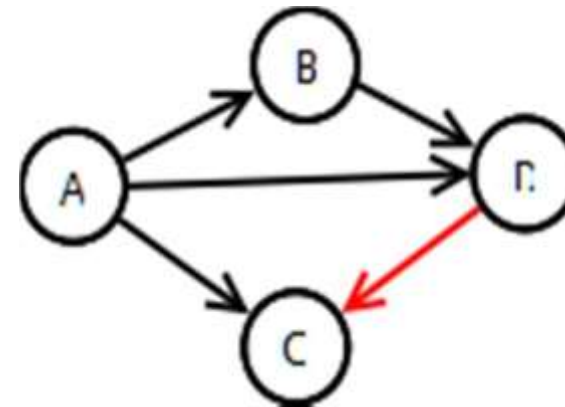
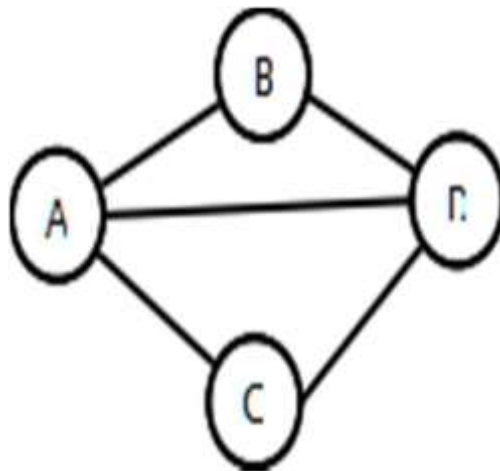




Basic Terminologies



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Basic Terminologies



Degree

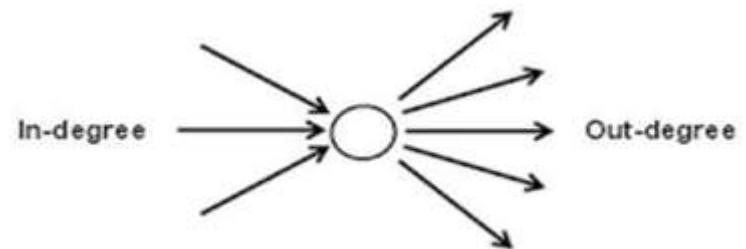
- Total number of edges connected to a vertex is said to be degree of that vertex.

Indegree

- Total number of incoming edges connected to a vertex is said to be indegree of that vertex.

Outdegree

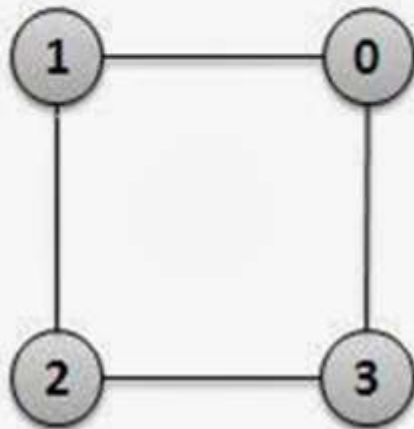
- Total number of outgoing edges connected to a vertex is said to be outdegree of that vertex.



Data Structures for Representing Graphs

Adjacency Matrix – Undirected Graph

- Adjacency matrix



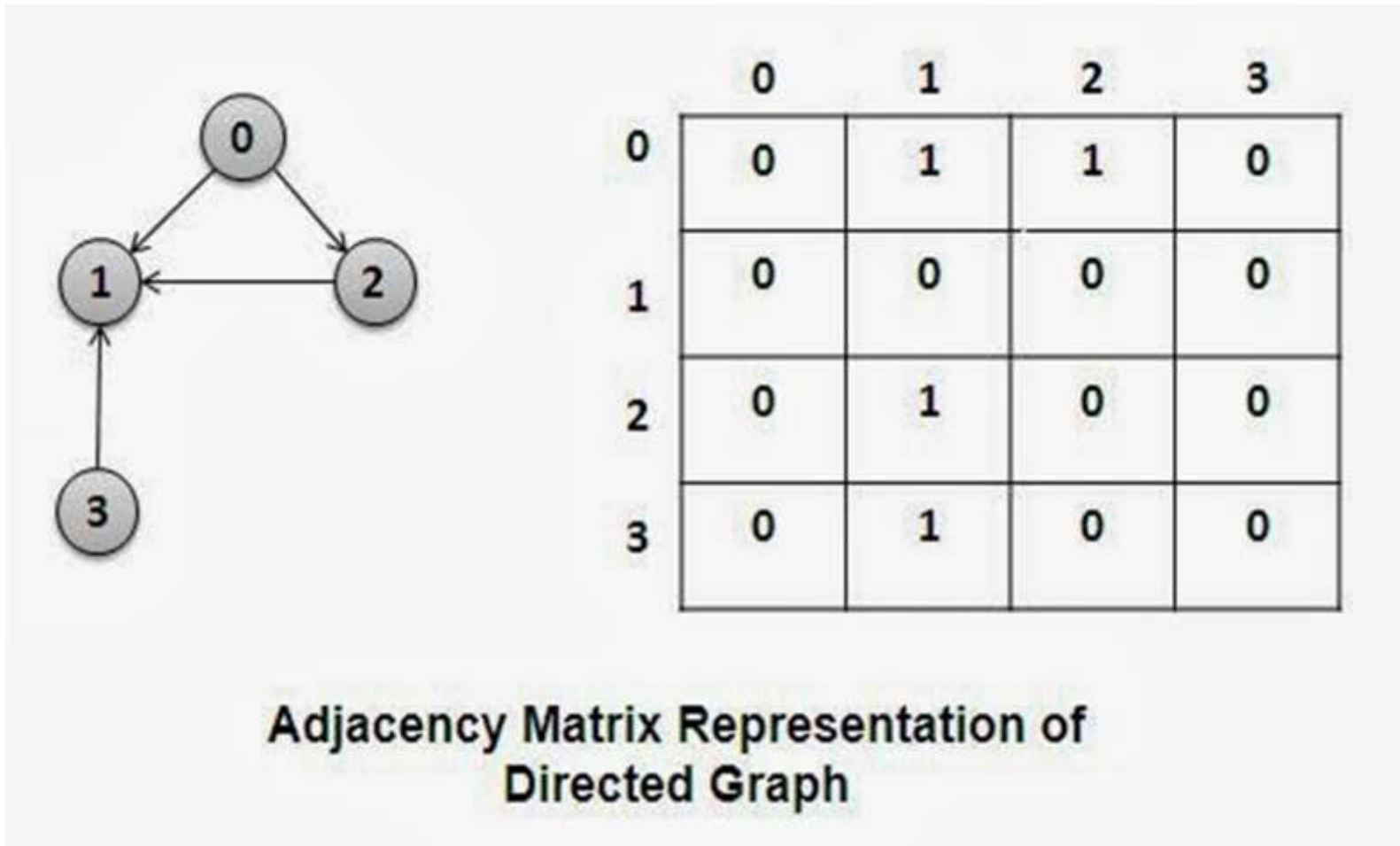
	0	1	2	3
0	0	1	0	1
1	1	0	1	0
2	0	1	0	1
3	1	0	1	0

Adjacency Matrix Representation of Undirected Graph



Data Structures for Representing Graphs

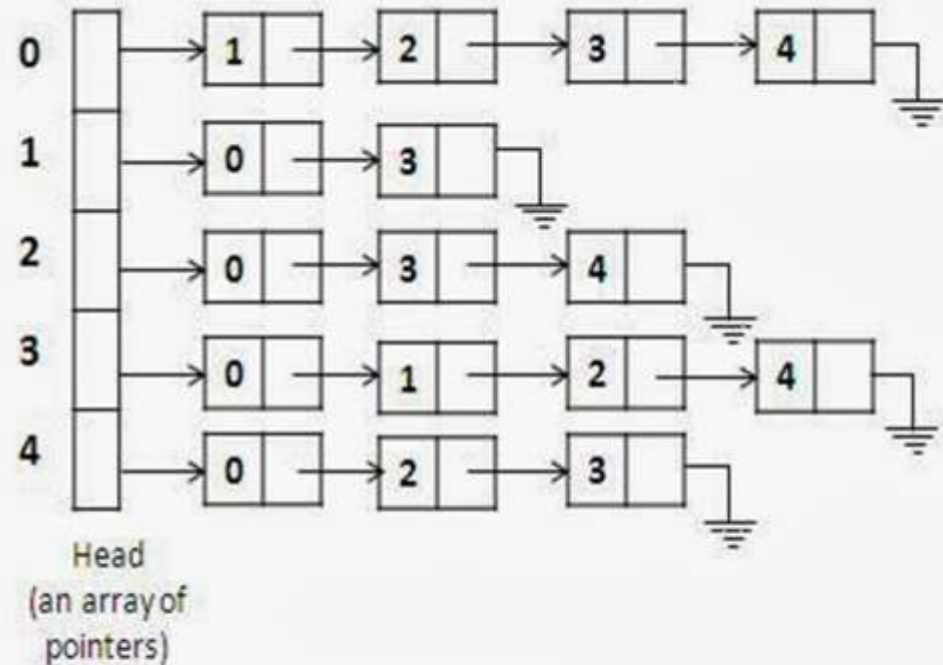
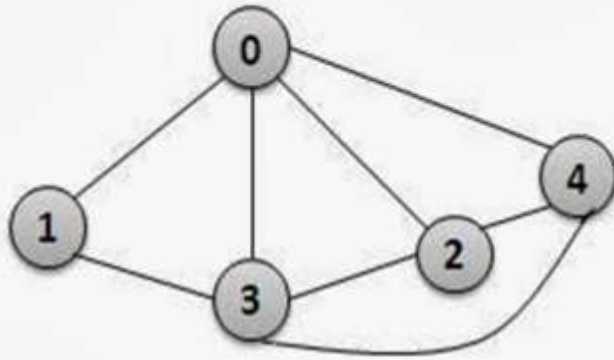
Adjacency Matrix – Directed Graph





Data Structures for Representing Graphs

Adjacency List



Adjacency List Representation of Graph

Summary of Graph
Introduction

Basic Terminologies

Representation of Graph

Adjacency Matrix

Adjacency List



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Recap

Graphs

- Basic Terminologies
- Representation of Graph



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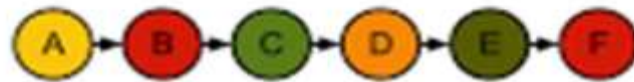
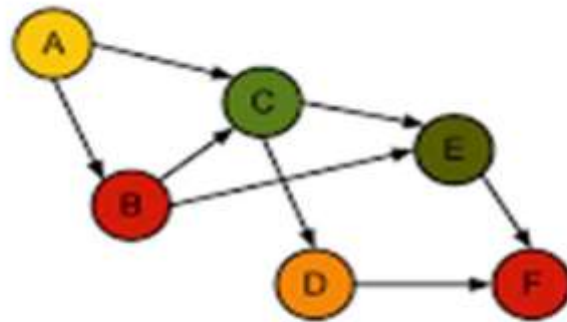
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Topological Sort

Linear ordering of Vertices in a directed acyclic graph such that if there is a path from V_i to V_j then V_j appears after V_i in the linear ordering

TOPOLOGICAL SORT



Steps in Topological Sort

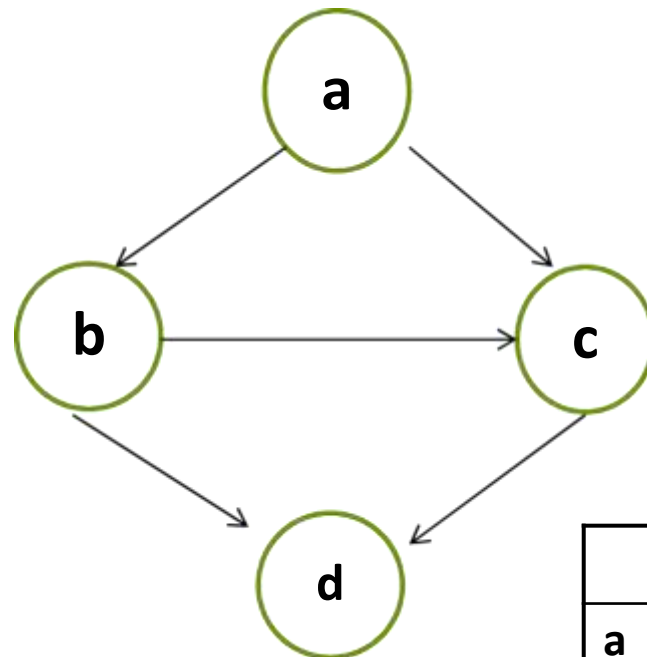
Step 1	Find the indegree for every vertex
Step 2	Place the vertices whose indegree is '0' on the empty queue
Step 3	Dequeue the vertex V and decrement the indegree's of all the adjacent vertices
Step 4	Enqueue the vertex on the queue,if its indegree falls to zero
Step 5	Repeat from step 3 until the queue becomes empty
Step 6	The topological ordering is the order in which the vertices dequeued

Routine for Topological Sort

```
Void Topsort(Graph G)
{
Queue Q;
Int counter=0;
Q=CreateQueue (Num Vertex)
Makeempty(Q);
For each vertex V
If (indegree[V]==0)
Enqueue(V,Q);
While(!IsEmpty(Q))
{
V=Dequeue(Q);
```

```
TopNum[V]=+counter;
For each W adjacent to V
IF(--Indegree[W]==0)
Enqueue(W,Q);
}
If(counter !=Num Vettex)
Error("Graph has a cycle");
DisposeQueue(Q);
}
```

Topological Sort - Example



	a	b	c	d
a	0	1	1	0
b	0	0	1	1
c	0	0	0	1
d	0	0	0	0

<p>Step 1</p>	<p>Indegree[a]=0 Indegree[b]=1 Indegree[c]=2 Indegree[d]=2</p>
<p>Step 2</p>	<p>Enqueue the Vertex,whose Indegree is '0' Vertex 'a' is 0,so place it on the queue</p>
<p>Step 3</p>	<p>Dequeue the vertex 'a' from the queue and decrement the indegree's of all the adjacent vertices 'b' & 'c' Hence, Indegree[b]=0 and Indegree[c]=1 Now,Enqueue the vertex 'b' as its indegree becomes zero</p>
<p>Step 4</p>	<p>Dequeue the vertex 'b' from Q and decrement the indegree's of its adjacent vertices 'c' & 'd' Hence, Indegree[c]=0 and Indegree[d]=1 Now,Enqueue the vertex 'c' as its indegree becomes</p>

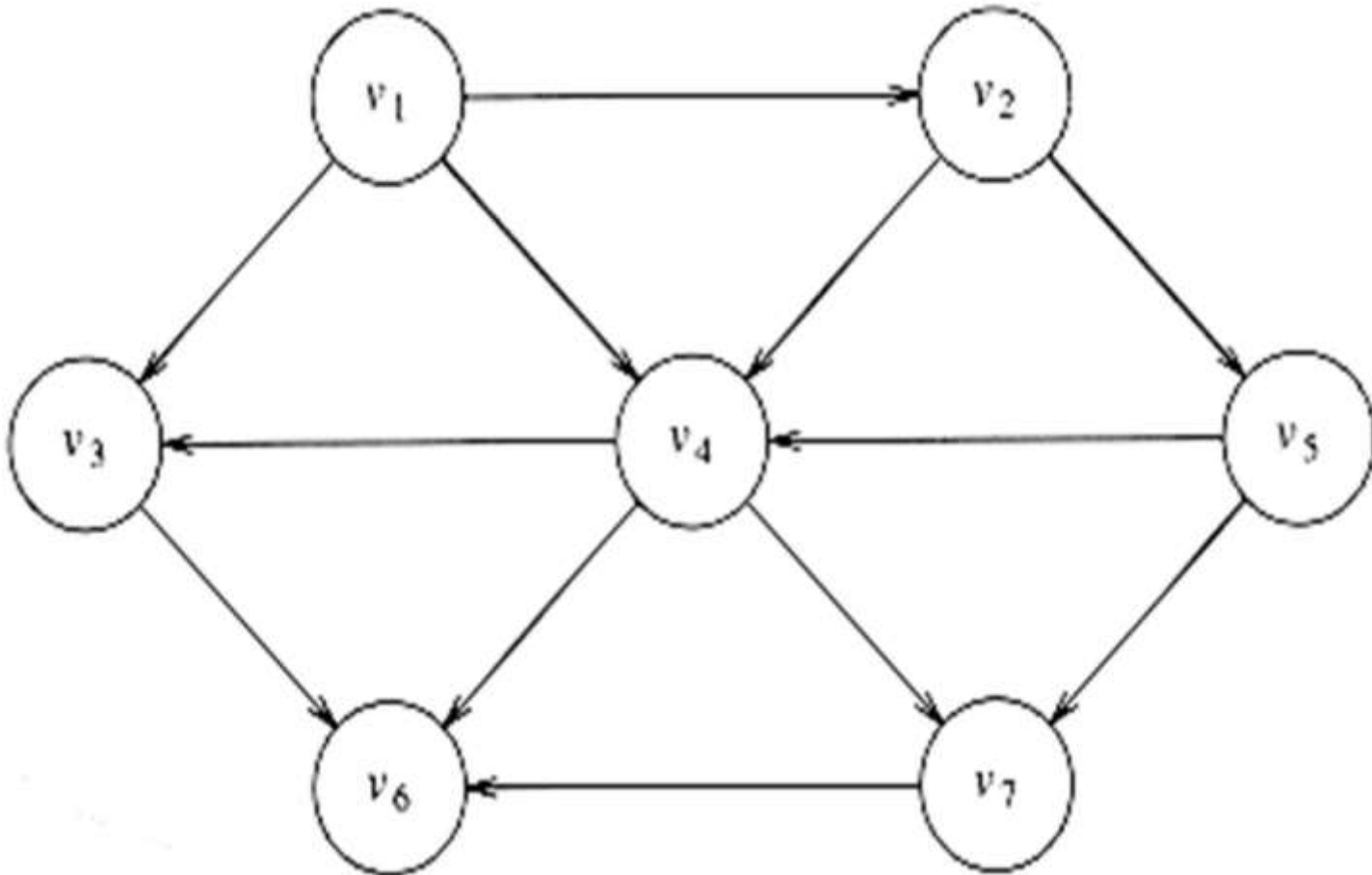
Steps in Topological Sort

Step 5	<p>Dequeue the vertex 'c' from Q and decrement the indegree's of its adjacent vertices 'd'</p> <p>Hence, Indegree[d]=0</p> <p>Now, Enqueue the vertex 'd' as its indegree becomes zero</p>
Step 6	<p>Dequeue the vertex 'd'</p>
Step 7	<p>As the queue becomes empty, topological ordering is performed , which is nothing but the order in which the vertices are dequeued</p>

Result of the Graph

Vertex	1	2	3	4
a	0	0	0	0
b	1	0	0	0
c	2	1	0	0
d	2	2	1	0
Enqueue	a	b	c	d
Dequeue	a	b	c	d

Example 2



Result of the Graph

Indegree Before Dequeue #

Vertex 1 2 3 4 5 6 7

v_1	0	0	0	0	0	0	0
v_2	1	0	0	0	0	0	0
v_3	2	1	1	1	0	0	0
v_4	3	2	1	0	0	0	0
v_5	1	1	0	0	0	0	0
v_6	3	3	3	3	2	1	0
v_7	2	2	2	1	0	0	0

enqueue $v_1 v_2 v_5 v_4 v_3 v_7 v_6$

dequeue $v_1 v_2 v_5 v_4 v_3 v_7 v_6$

•

Summary of Topological Sort

Linear Ordering of Vertices

Find the Indegree of all vertices

Topological sort algorithm

Topological ordering of vertices