# SNS COLLEGE OF TECHNOLOGY

**Coimbatore-35**

**An Autonomous Institution**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## COURSE NAME : 23ITT201 DATA STRUCTURES

## II YEAR/ III SEMESTER

### UNIT – IV MULTIWAY SEARCH TREES AND GRAPH

## Topic: *GRAPH*

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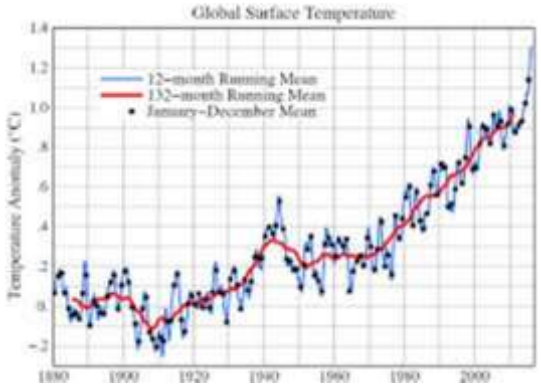
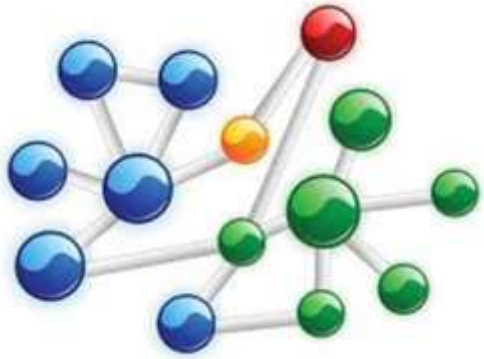



## *UNIT IV*

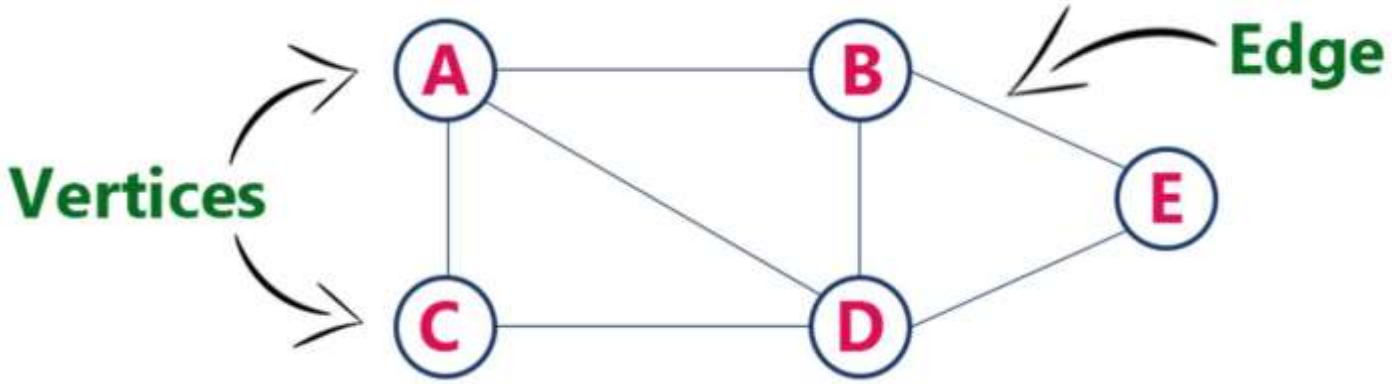



# *Introduction*

- **Graph** is a collection of vertices and arcs which connects vertices in the graph
- Graph is a collection of nodes and edges which connects nodes in the graph
- Generally, a graph **G** is represented as **G = ( V , E )**, where **V is set of vertices** and **E is set of edges**.

# *Introduction*

The following is a graph with 5 vertices and 6 edges.
This graph G can be defined as G = ( V , E )
Where V = {A,B,C,D,E} and E = {(A,B),(A,C)(A,D),(B,D),(C,D),(B,E),(E,D)}.



# *Basic Terminologies*

**Vertex**

A individual data element of a graph is called as Vertex. **Vertex** is also known as **node**. In above example graph, A, B, C, D & E are known as vertices

**Edge**

An edge is a connecting link between two vertices. **Edge** is also known as **Arc**. An edge is represented as (starting Vertex, ending Vertex).

The link between vertices A and B is represented as (A,B). In above example graph, there are 7 edges (i.e., (A,B), (A,C), (A,D), (B,D), (B,E), (C,D), (D,E)).

# *Basic Terminologies*

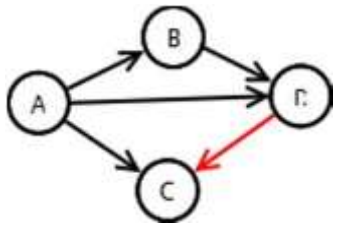
- **Undirected Edge -** An undirected edge is a bidirectional edge. If there is a undirected edge between vertices A and B then edge (A , B) is equal to edge (B , A).
- **Directed Edge -** A directed edge is a unidirectional edge. If there is a directed edge between vertices A and B then edge (A , B) is not equal to edge (B , A).
- **Weighted Edge -** A weighted egde is an edge with cost on it.

**Undirected Graph**

- A graph with only undirected edges is said to be undirected graph.

**Directed Graph**

- A graph with only directed edges is said to
- be directed graph.



# *Basic Terminologies*

**Mixed Graph**

- A graph with undirected and directed edges is said to be mixed graph.

**End vertices or Endpoints**

- The two vertices joined by an edge are called the end vertices (or endpoints) of the edge.

**Origin**

- If an edge is directed, its first endpoint is said to be origin of it.

**Destination**

- If an edge is directed, its first endpoint is said to be origin of it and the other endpoint is said to be the destination of the edge.

# *Basic Terminologies*

**Adjacent**

If there is an edge between vertices A and B then both A and B are said to be adjacent. In other words, Two vertices A and B are said to be adjacent if there is an edge whose end vertices are A and B.

**Incident**

An edge is said to be incident on a vertex if the vertex is one of the endpoints of that edge.

**Outgoing Edge**

A directed edge is said to be outgoing edge on its orign vertex.

**Incoming Edge**

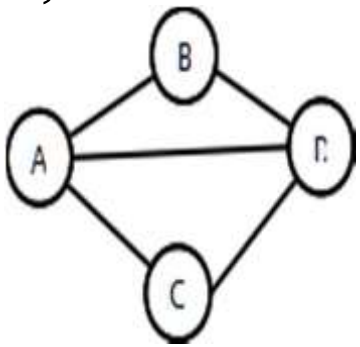
A directed edge is said to be incoming edge on its destination vertex.

# *Basic Terminologies*

- **Degree of a Node**
  - In-degree: Number of edges pointing **to** a node
  - Out-degree: Number of edges pointing **from** a node
- **Path**: sequence of vertices in which each pair of successive vertices is connected by an edge
  **Cycle**: a path that starts and ends on the same vertex
  **Simple path**: a path that does not cross itself That is, no vertex is repeated (except first and last)

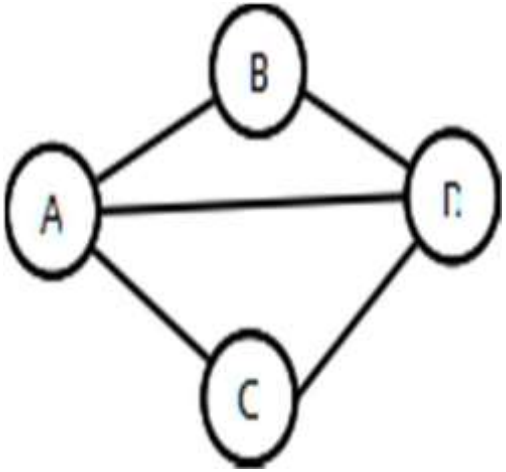
Simple paths cannot contain cycles

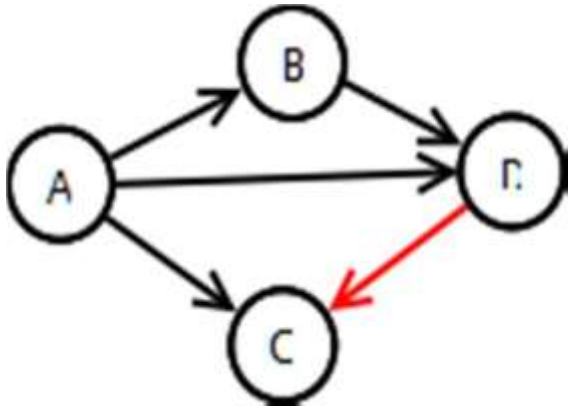
**Length** of a path: Number of edges in the path Sometimes the sum of the weights of the edges



# *Basic Terminologies*

- A **Cyclic** graph contains cycles Example: roads (normally)
  An **acyclic** graph contains no cycles



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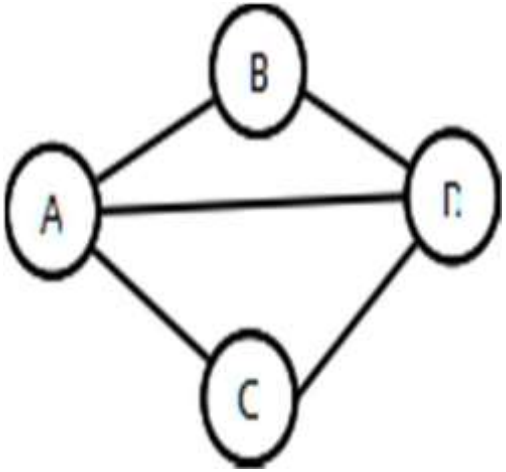


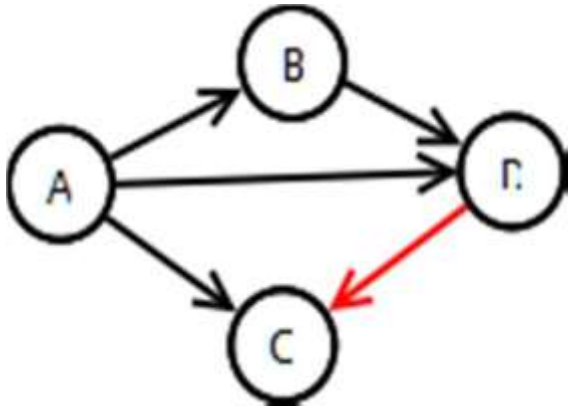

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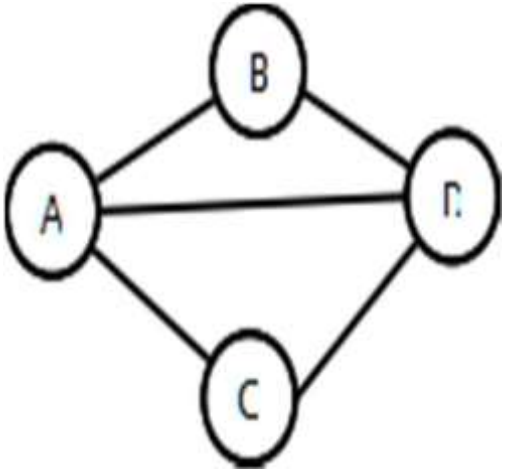


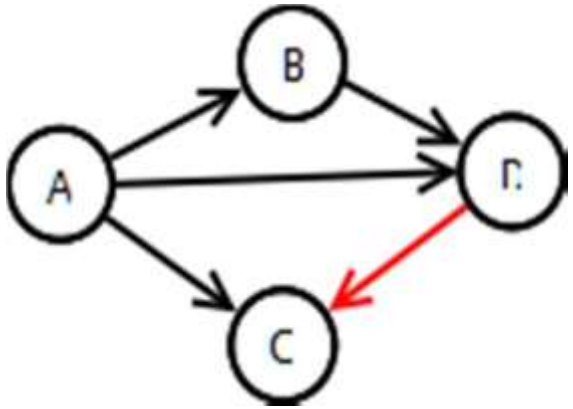

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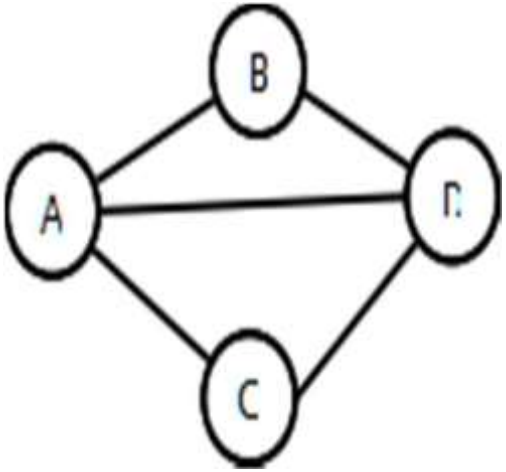


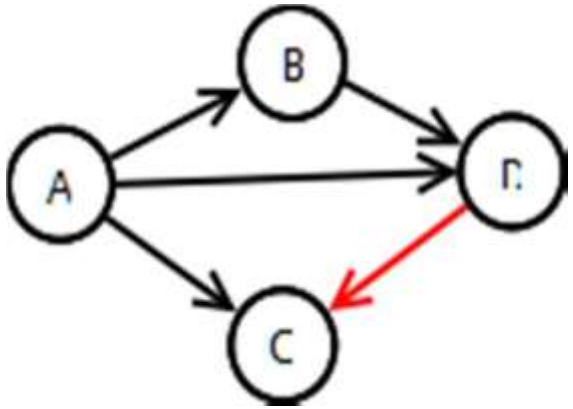

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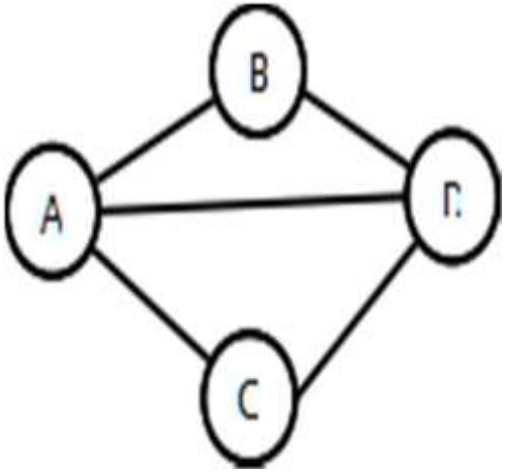


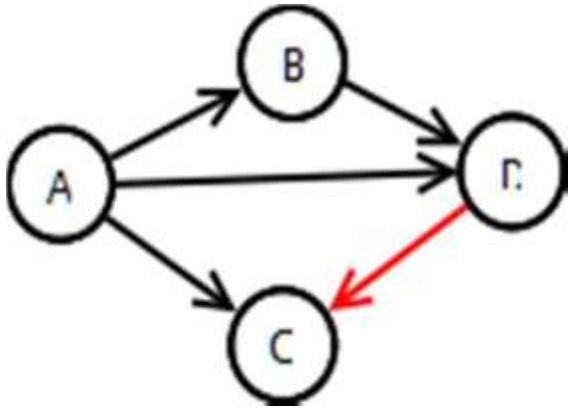

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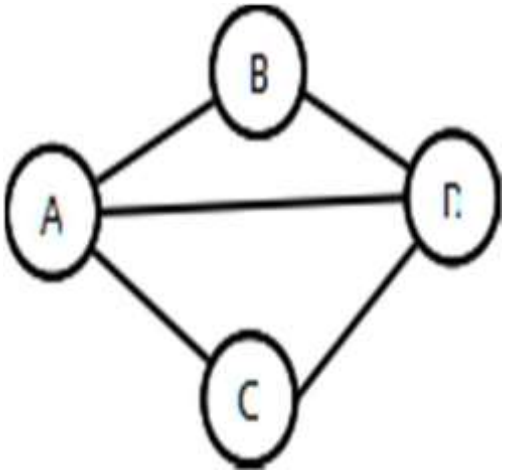
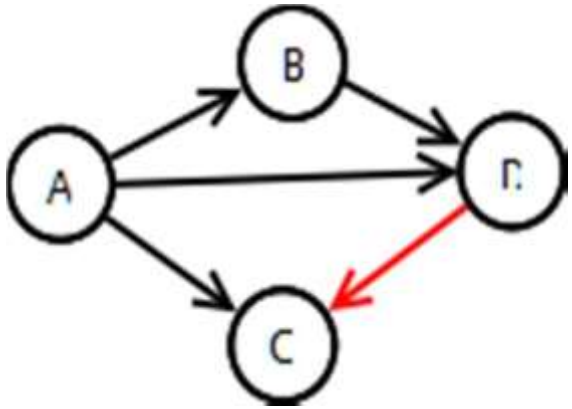


# *Basic Terminologies*

**Degree**

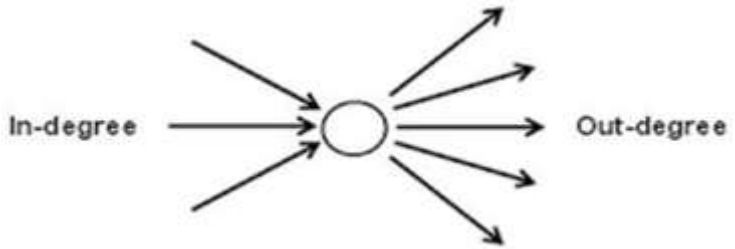
- Total number of edges connected to a vertex is said to be degree of that vertex.

**Indegree**

- Total number of incoming edges connected to a vertex is said to be indegree of that vertex.

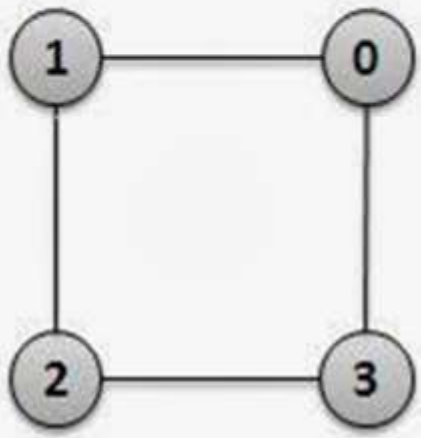
**Outdegree**

- Total number of outgoing edges connected to a vertex is said to be outdegree of that vertex.



# *Data Structures for Representing Graphs Adjacency Matrix –Undirected Graph*

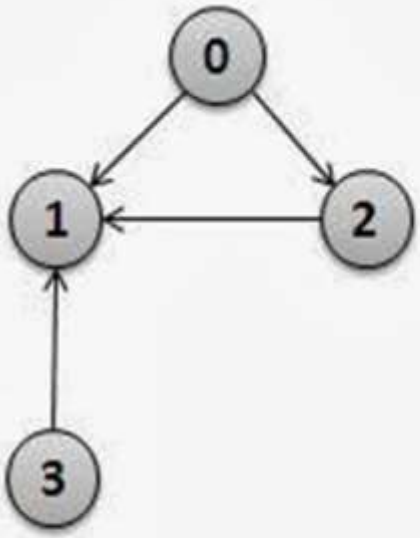
- Adjacency matrix




| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 2 | 0 | 1 | 0 | 1 |
| 3 | 1 | 0 | 1 | 0 |

**Adjacency Matrix Representation of Undirected Graph**

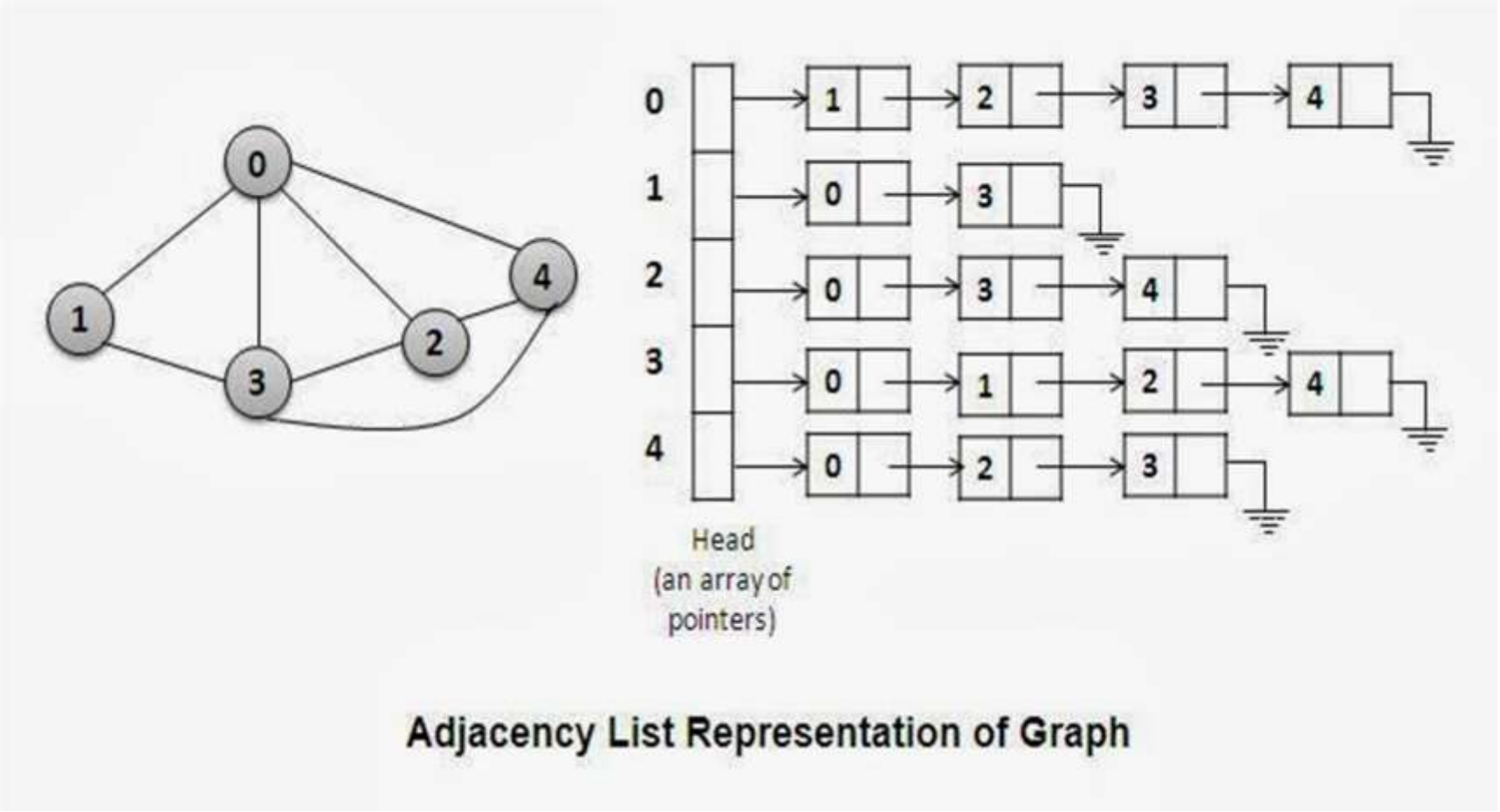
# *Data Structures for Representing Graphs*
## *Adjacency Matrix –Directed Graph*



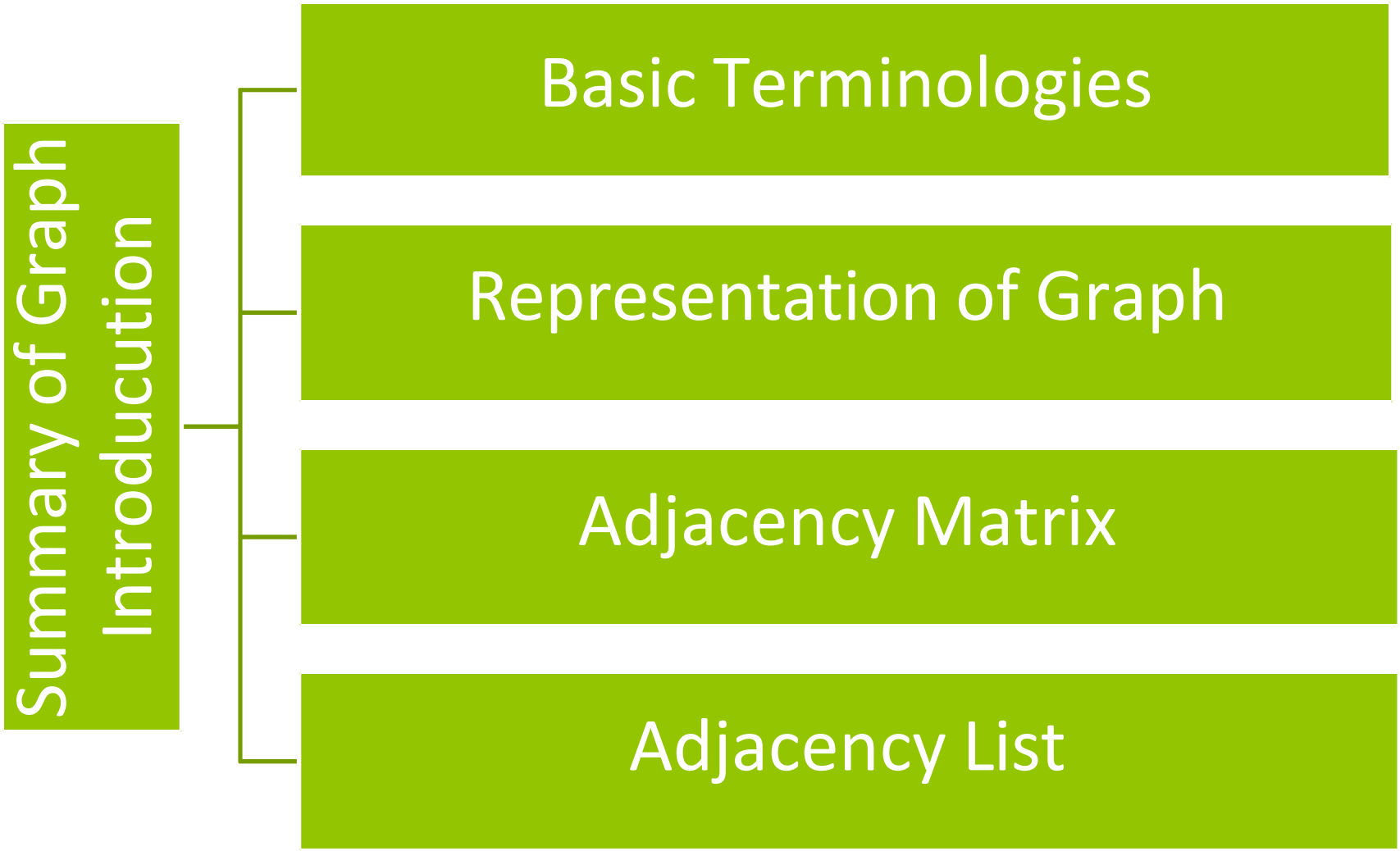

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 0 | 0 |

**Adjacency Matrix Representation of Directed Graph**

# *Data Structures for Representing Graphs Adjacency List*




**Adjacency List Representation of Graph**


Summary of Graph Introducution
Basic Terminologies
Representation of Graph
Adjacency Matrix
Adjacency List

# *Recap*

## *Graphs*

- Basic Terminologies
- Representation of Graph

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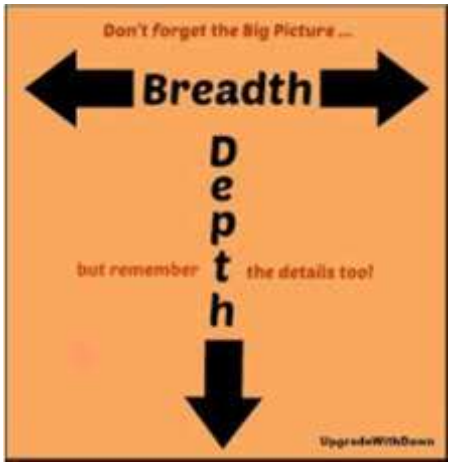
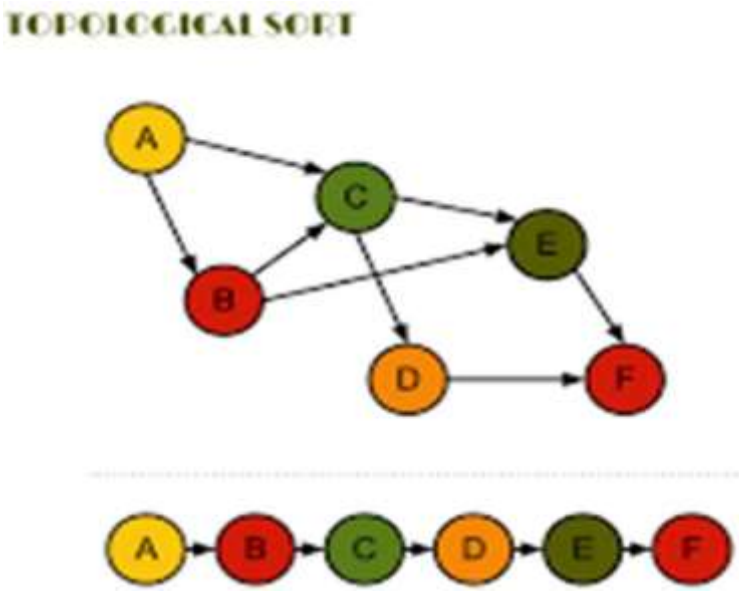




# *Topological Sort*

Linear ordering of Vertices in a directed acyclic graph such that if there is a path from Vi to Vj then Vj appears after Vi in the linear ordering



# *Steps in Topological Sort*

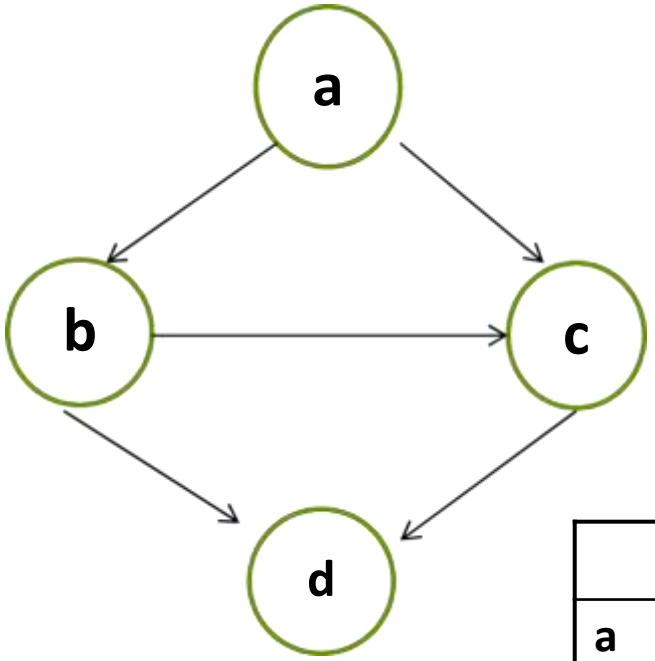
| | |
|---|---|
| **Step 1** | Find the indegree for every vertex |
| **Step 2** | Place the vertices whose indegree is ‘0’ on the empty queue |
| **Step 3** | Dequeue the vertex V and decrement the indegree’s of all the adjacent vertices |
| **Step 4** | Enqueue the vertex on the queue,if its indegree falls to zero |
| **Step 5** | Repeat from step 3 until the queue becomes empty |
| **Step 6** | The topological ordering is the order in which the vertices dequeued |
| | |

# *Routine for Topological Sort*

```
Void Topsort(Graph G)
{
Queue Q;
Int counter=0;
Q=CreateQueue (Num Vertex)
Makeempty(Q);
For each vertex V
If (indegree[V]==0)
Enqueue(V,Q);
While(!IsEmpty(Q))
{
V=Dequeue(Q);
```

```
TopNum[V]=+counter;
For each W adjacent to V
IF(--Indegree[W]==0
Enqueue(W,Q);
}
If(counter !=Num Vettex)
Error("Graph has a cycle");
DisposeQueue(Q);
}
```

# *Topological Sort - Example*




| | a | b | c | d |
|---|---|---|---|---|
| a | 0 | 1 | 1 | 0 |
| b | 0 | 0 | 1 | 1 |
| c | 0 | 0 | 0 | 1 |
| d | 0 | 0 | 0 | 0 |

| | |
|---|---|
| **Step 1** | Indegree[a]=0 Indegree[b]=1<br>Indegree[c]=2 Indegree[d]=2 |
| **Step 2** | Enqueue the Vertex,whose Indegree is'0'<br>Vertex 'a' is 0,so place it on the queue |
| **Step 3** | Dequeue the vertex 'a' from the queue and decrement the indegree's of all the adjacent vertices'b' & 'c'<br>Hence, **Indegree[b]=0 and Indegree[c]=1**<br>Now,Enqueue the vertex 'b' as its indegree becomes zero |
| **Step 4** | Dequeue the vertex 'b' from Q and decrement the indegree's of its adjacent vertices'c' & 'd'<br>Hence**, Indegree[c]=0 and Indegree[d]=1**<br>Now,Enqueue the vertex 'c' as its indegree becomes |

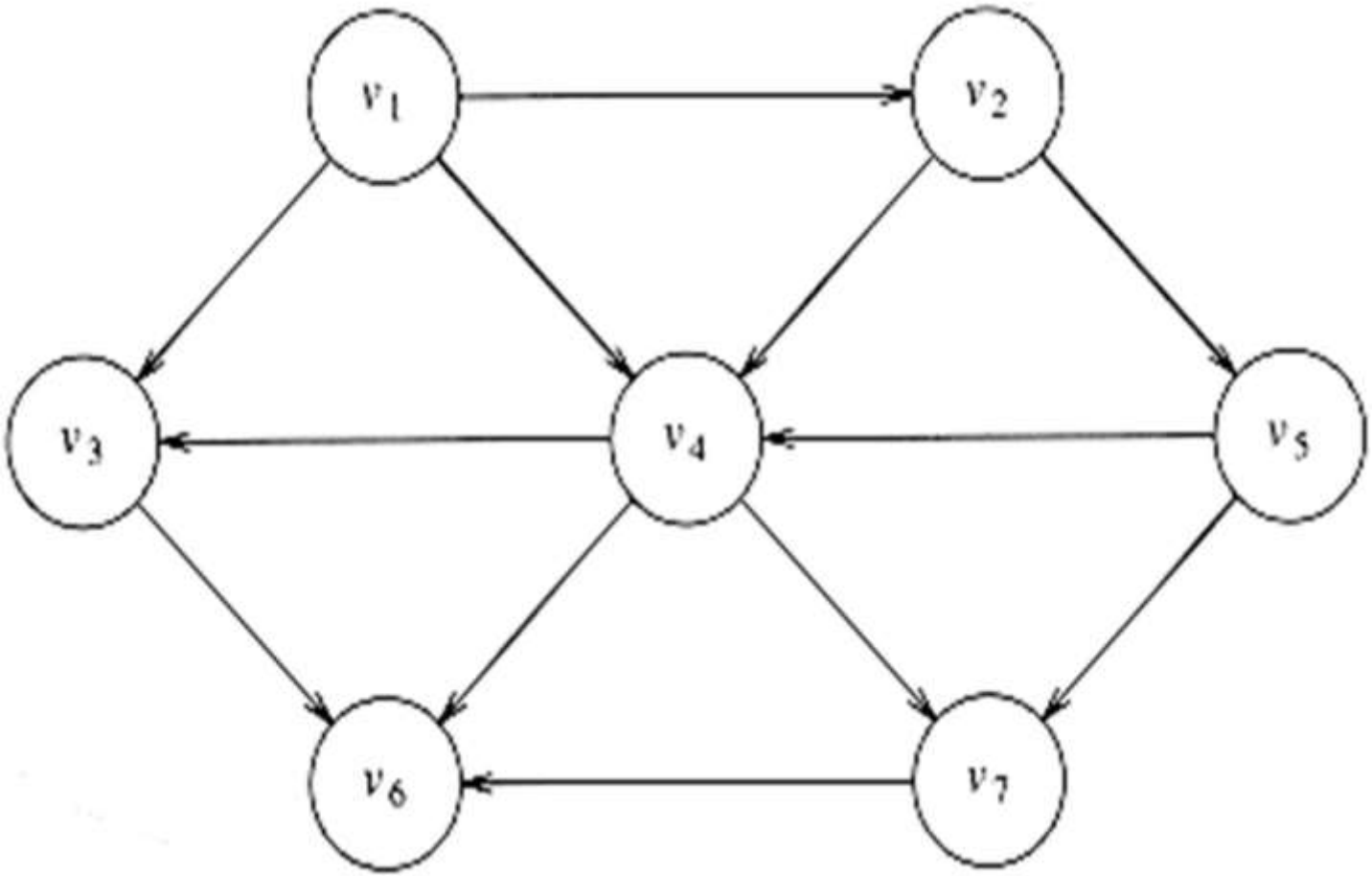
# *Steps in Topological Sort*

| | |
|---|---|
| **Step 5** | Dequeue the vertex ‘c’ from Q and decrement the indegree’s of its adjacent vertices’d’<br>Hence, **Indegree[d]=0**<br>Now,Enqueue the vertex ‘d’ as its indegree becomes zero |
| **Step 6** | Dequeue the vertex ‘d’ |
| **Step 7** | As the queue becomes empty, topological ordering is performed , which is nothing but the order in which the vertices are dequeued |

# *Result of the Graph*

| Vertex | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| a | **0** | 0 | 0 | 0 |
| b | 1 | **0** | 0 | 0 |
| c | 2 | 1 | **0** | 0 |
| d | 2 | 2 | 1 | **0** |
| Enqueue | a | b | c | d |
| Dequeue | a | b | c | d |

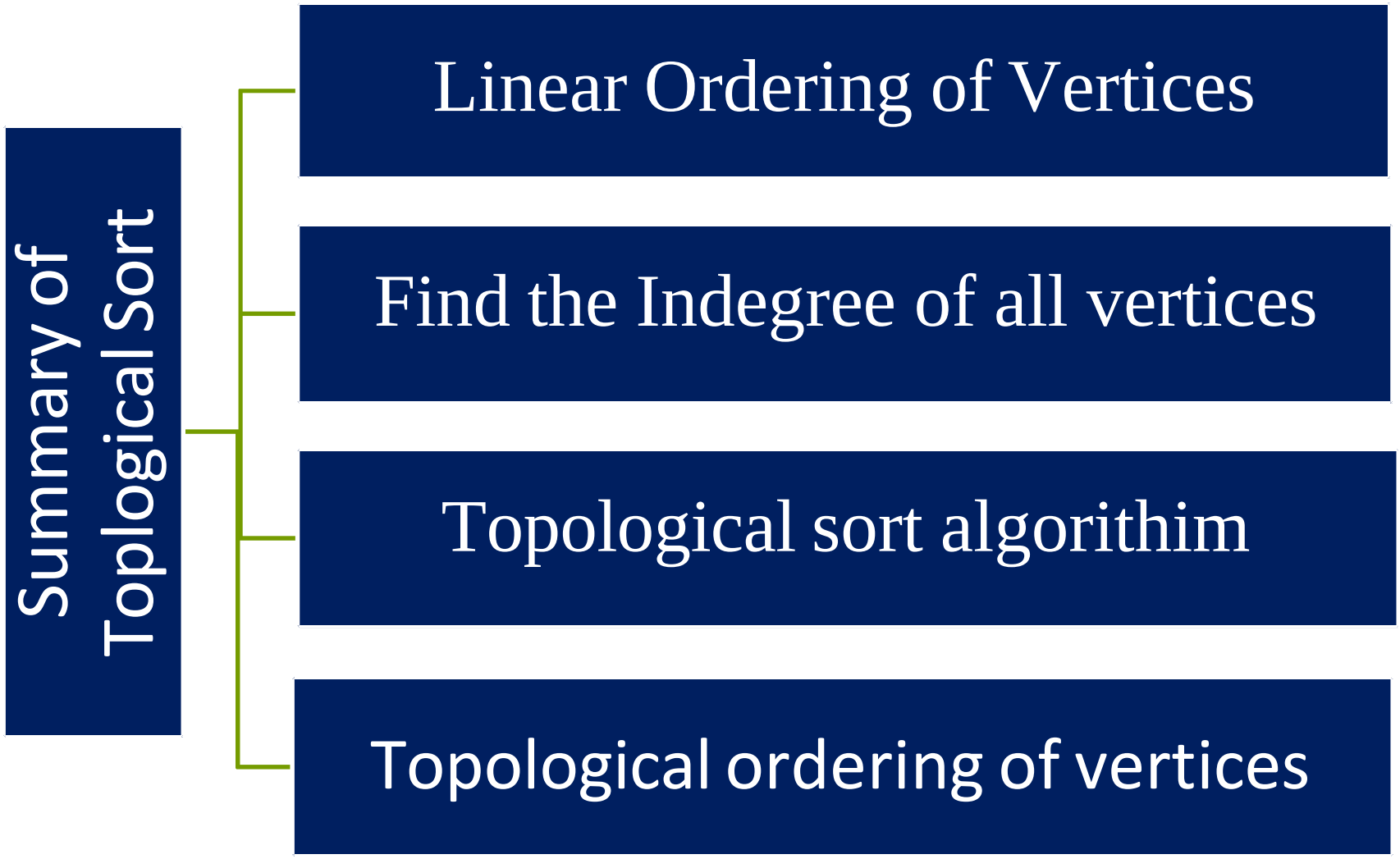
# *Example 2*



## *Result of the Graph*

| Indegree Before Dequeue # | | | | | | | |
|---|---|---|---|---|---|---|---|
| Vertex | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $v_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_2$ | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_3$ | 2 | 1 | 1 | 1 | 0 | 0 | 0 |
| $v_4$ | 3 | 2 | 1 | 0 | 0 | 0 | 0 |
| $v_5$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $v_6$ | 3 | 3 | 3 | 3 | 2 | 1 | 0 |
| $v_7$ | 2 | 2 | 2 | 1 | 0 | 0 | 0 |
| *enqueue* | $v_1$ | $v_2$ | $v_5$ | $v_4$ | $v_3$ | $v_7$ | $v_6$ |
| *dequeue* | $v_1$ | $v_2$ | $v_5$ | $v_4$ | $v_3$ | $v_7$ | $v_6$ |

•


Summary of Toplogical Sort
Linear Ordering of Vertices
Find the Indegree of all vertices
Topological sort algorithim
Topological ordering of vertices