SNS COLLEGE OF TECHNOLOGY

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Department of Biomedical Engineering

Course Name: 23BMT201 & Circuit Analysis

I Year : II Semester

Unit V –RESONANCE CIRCUITS & COUPLED CIRCUITS

Topic : Frequency Response

When Resonance Occurs

- Resonance occurs in any circuit that has energy storage elements, at least one inductor and one capacitor.
- Under resonance, the total impedance is equal to the resistance only and maximum power is drawn from the supply by the circuit.
- Under resonance, the total supply voltage and supply current are in phase. So, the power factor (PF) becomes unity.
- At resonance, *L* and *C* elements exchange energy freely as a function of time, which results in sinusoidal oscillations either across *L* or *C*.
- **TYPES OF RESONANCE**
  - Series resonance.
  - Parallel resonance

Applications of Resonance

- Resonant circuits (series or parallel) are used in many applications such as selecting the desired stations in radio and TV receivers.
- Most common applications of resonance are based on the frequency dependent response. (“tuning” into a particular frequency/channel)
- A series resonant circuit is used as voltage amplifier.
- A parallel resonant circuit is used as current amplifier.
- A resonant circuit is also used as a filter.

Resonance in Series RLC Circuit

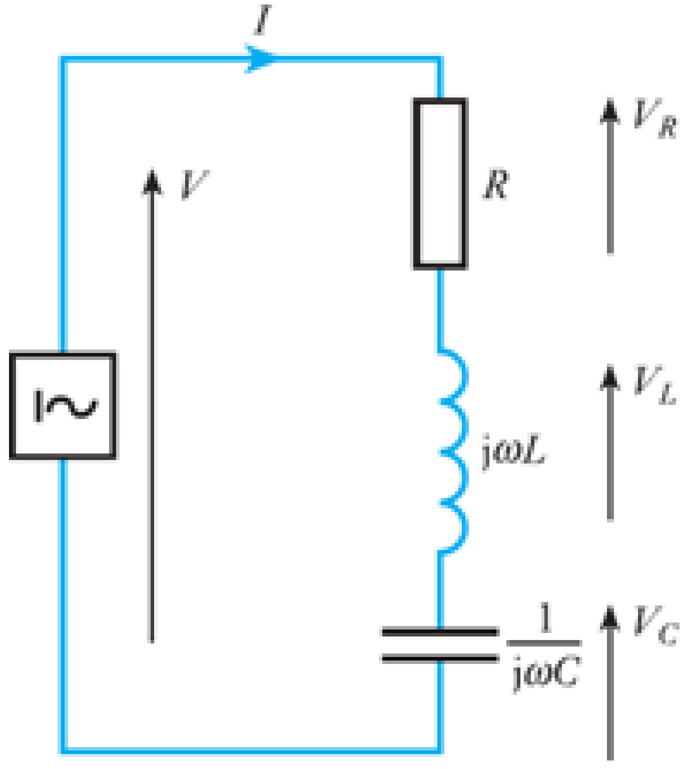
Resonance is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

The input impedance is as follows,

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

At resonance, the net reactance becomes zero. Therefore,

$$\omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \text{ rad/s};\ f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$



Series resonant *RLC* circuit

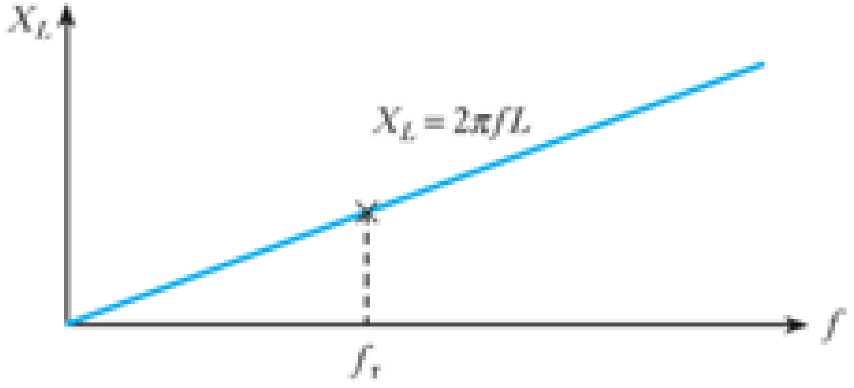
where ω_r and f_r represent resonant frequency in rad/s and in Hz, respectively

REACTANCE (X_L, X_C) VS FREQUENCY PLOTS

The value of the reactance X of the circuit is,

$$X = \omega L - \frac{1}{\omega C}$$ → Depends on frequency

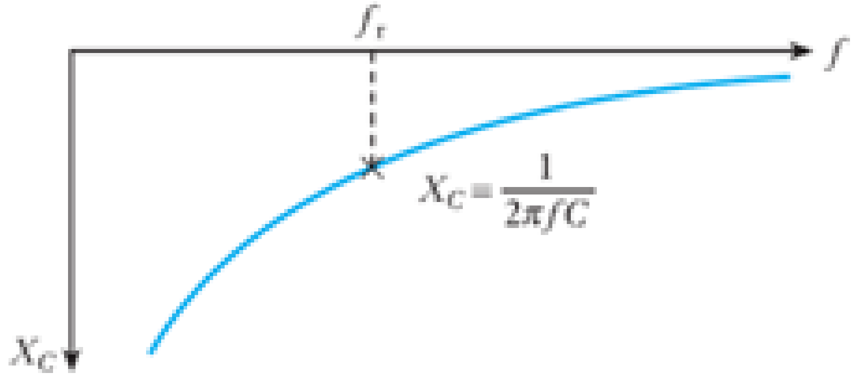
Variation of inductive reactance with frequency



The inductive reactance:

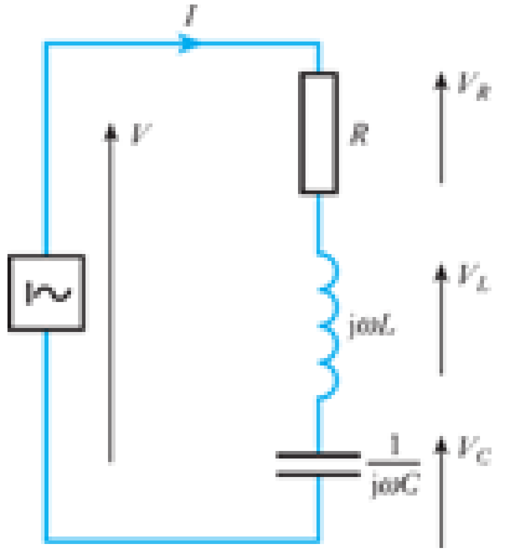
$$|X_L| = \omega L = 2\pi f L$$ → Increases linearly with frequency

Variation of Capacitive reactance with frequency



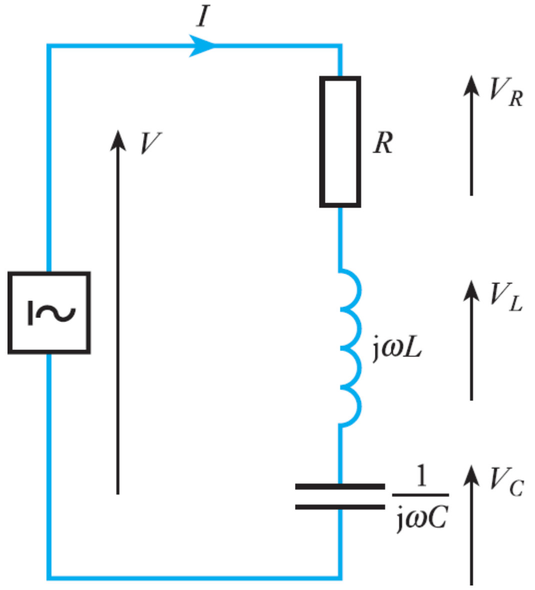
The capacitive reactance:

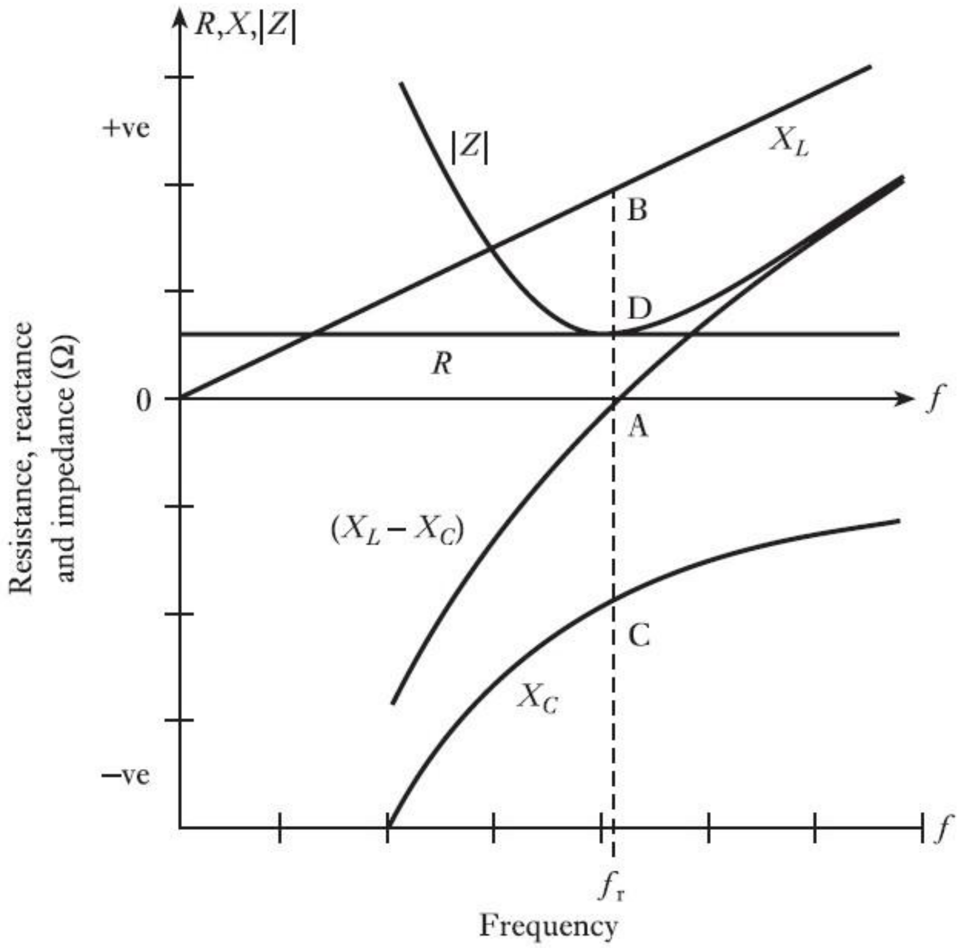
$$|X_C| = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$ → Decreases with frequency and it is largest at low frequencies



Variation of Reactance and Impedance with Frequency

- **At resonant frequency f_r**, $|Z| = R$, the power factor is unity (purely resistive).
- **Below f_r**, $|X_L| < |X_C|$, so the circuit is more capacitive and the power factor is leading.
- **Above f_r**, $|X_L| > |X_C|$, so the circuit is more inductive and the power factor is lagging.

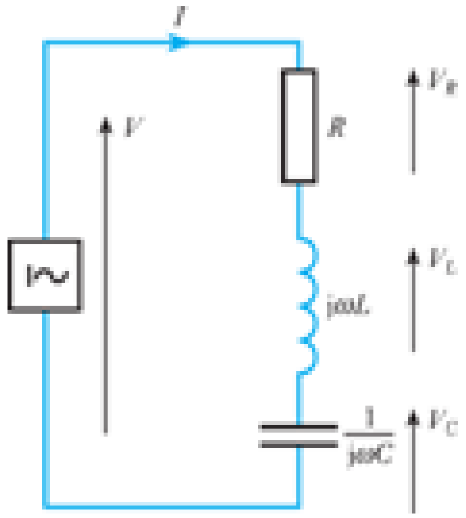


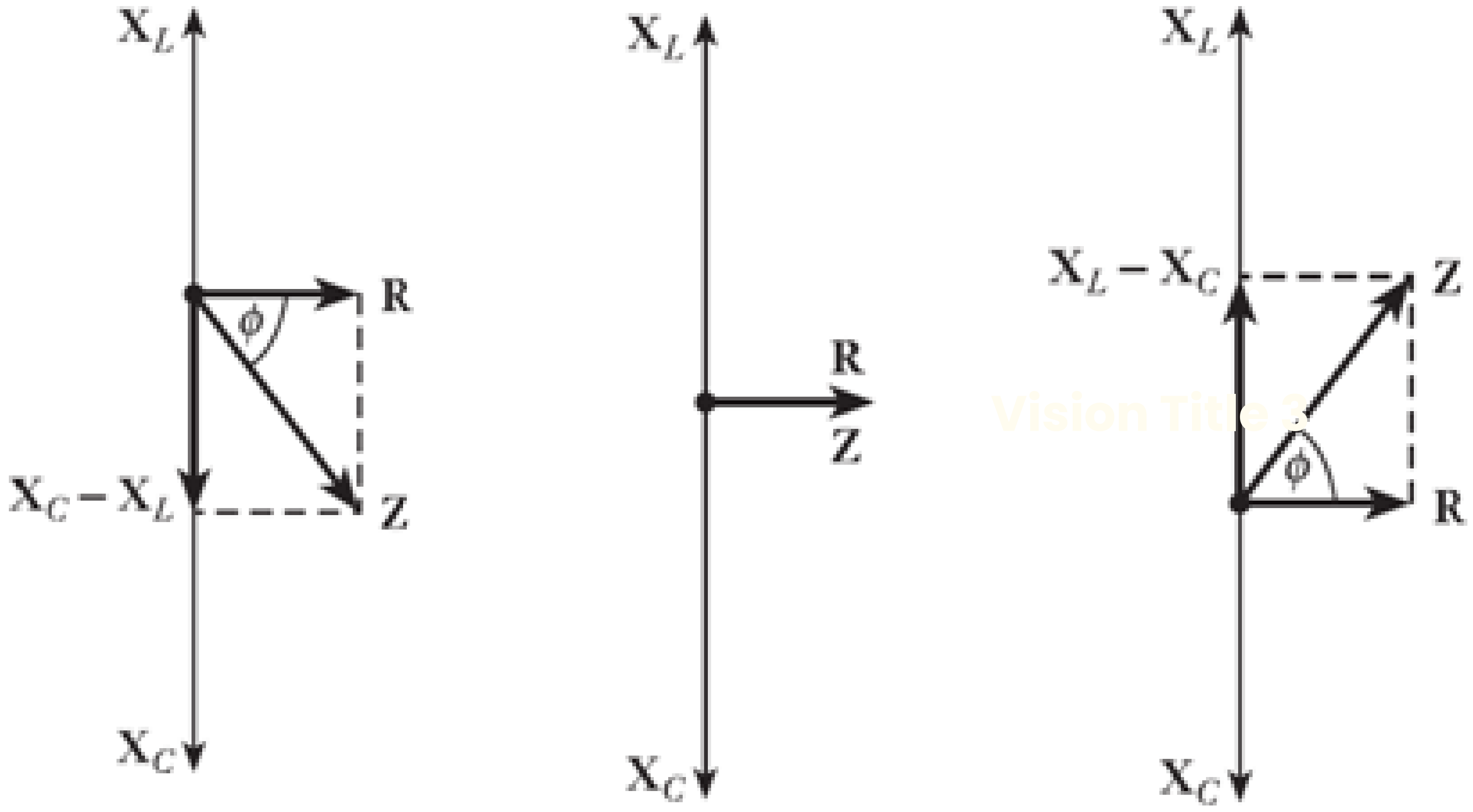


Impedance Phasor Diagrams

The phase of the circuit impedance is given by

$$\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$$





Below f_r, $X_C > X_L$ — At f_r, $X_C = X_L$, $Z = R$ — Above f_r, $X_L > X_C$

- Below f_r, $X_L < X_C$, ϕ is negative, the circuit is capacitive.
- At resonance (f_r), $X_L = X_C$, ϕ is zero, the circuit is purely resistive.
- Above f_r, $X_L > X_C$, ϕ is positive, the circuit is inductive.

The Current in a Series RLC Circuit

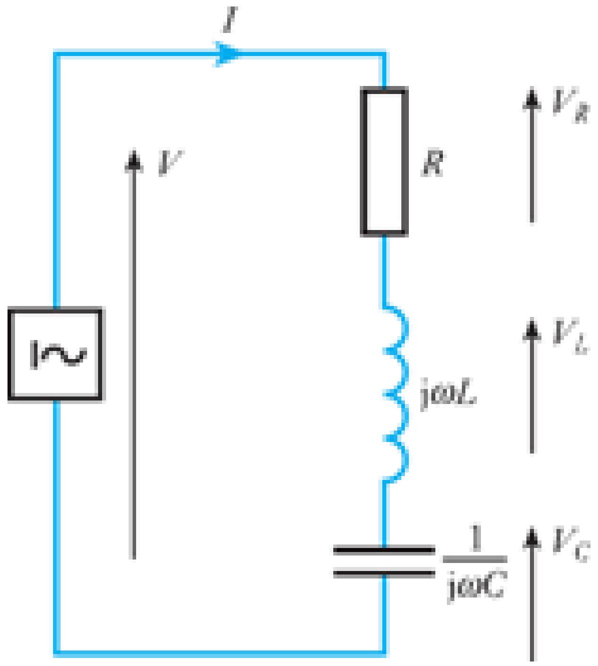
The circuit current is given by

$$I = \frac{V}{Z\angle\phi} = \frac{V\angle -\phi}{Z}$$

$$I = \frac{V}{\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}}\angle -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

The current is maximum when $\omega L = 1/(\omega C)$, when the circuit is resistive ($\phi = 0$). Therefore,

$$I_m = \frac{V}{R}$$



Variation of Magnitude and Phase of Current with Frequency

- The current is maximum at resonant frequency (f_r).

Variation of magnitude $|I|$ and phase current with frequency in a series RL circuit