SNS COLLEGE OF TECHNOLOGY

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Department of Biomedical Engineering

Course Name: 23BMT201 & Circuit Analysis

I Year : II Semester

Unit V –RESONANCE CIRCUITS & COUPLED CIRCUITS

Topic : Q Factor & Bandwidth

Quality Factor (Q)

- The "sharpness" of the resonance in a resonant circuit is measured quantitatively by the quality factor Q.
- The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:

$$Q = 2\pi \left(\frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} \right)$$

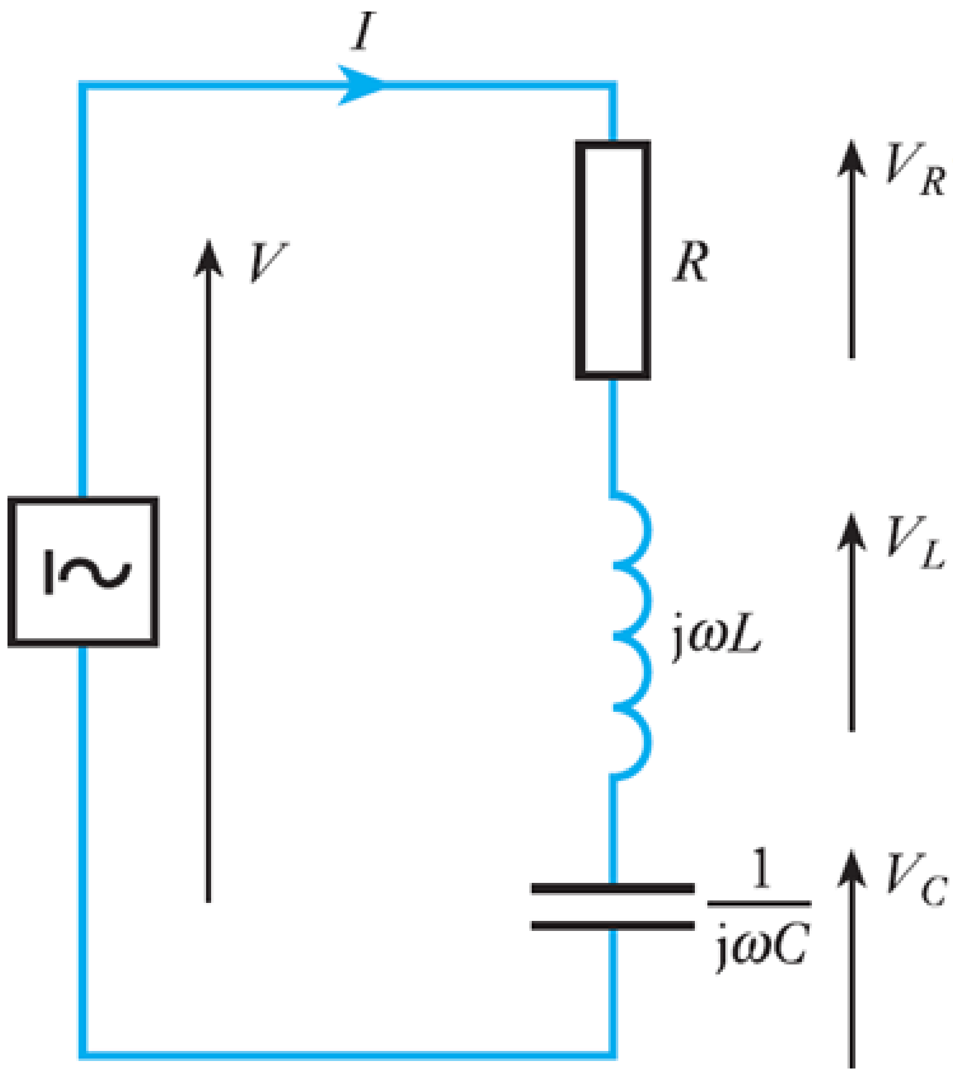
- It is also regarded as a measure of the energy storage property of a circuit in relation to its energy dissipation property.

Quality Factor (Q)

- In the series *RLC* circuit, the quality factor (*Q*) is,

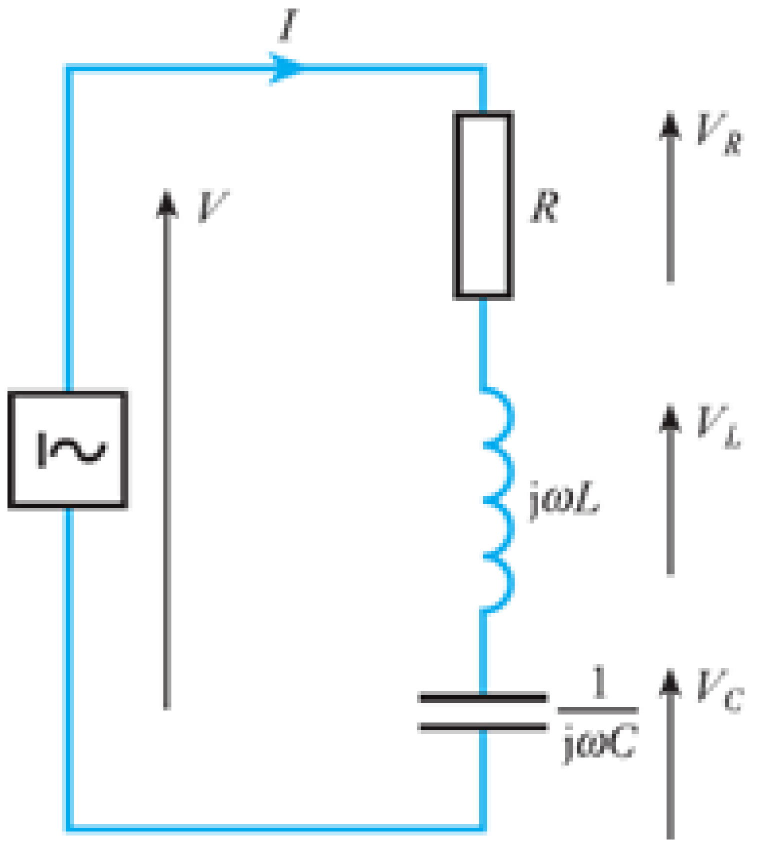
$$Q = 2\pi\left(\frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R\left(\frac{1}{f_r}\right)}\right) = \frac{2\pi f_r L}{R}$$

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Quality Factor (Q)

- The Q factor is also defined as the ratio of the reactive power, of either the capacitor or the inductor to the average power of the resistor at resonance: $$Q = \left(\frac{\text{Reactive power}}{\text{Average power}}\right)$$



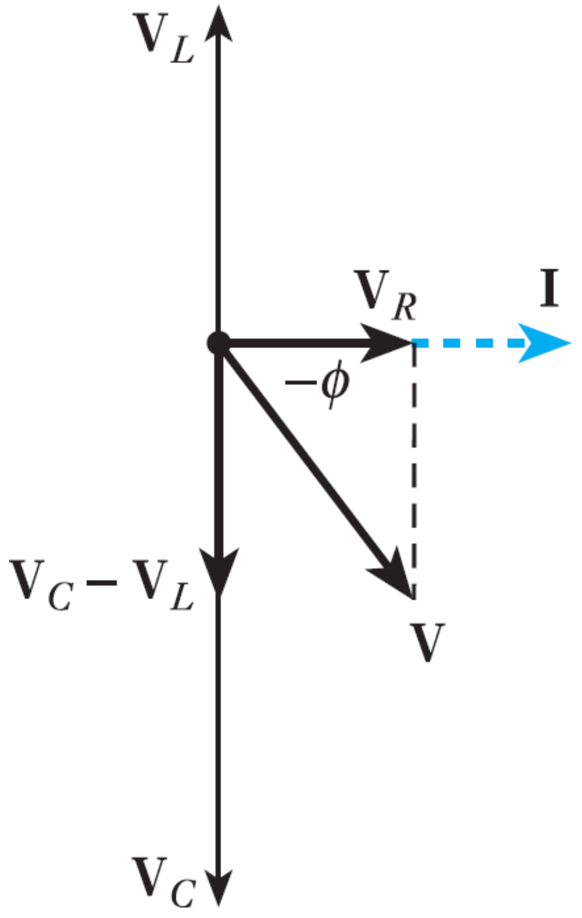
- For inductive reactance X_L at resonance:

$$Q = \left(\frac{\text{Reactive power}}{\text{Average power}}\right) = \frac{I^2 X_L}{I^2 R} = \frac{\omega L}{R}$$

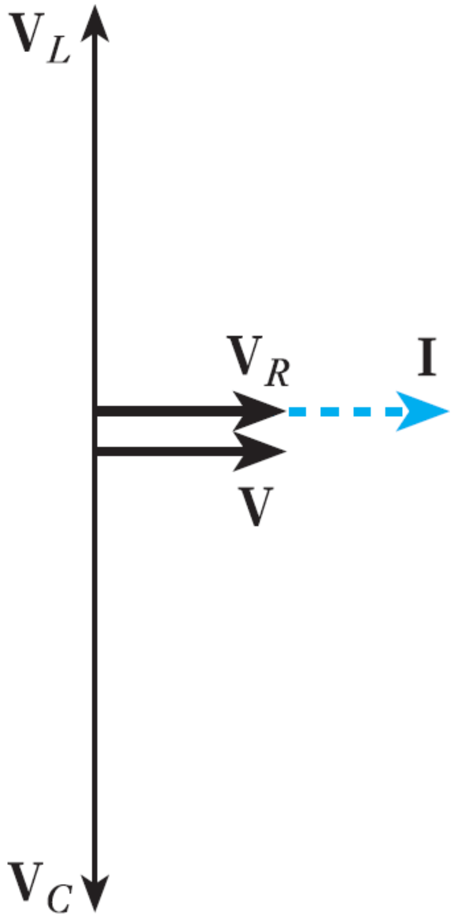
- For capacitive reactance X_L at resonance:

$$Q = \left(\frac{\text{Reactive power}}{\text{Average power}}\right) = \frac{I^2 X_C}{I^2 R} \quad \frac{1}{\omega_r CR}$$

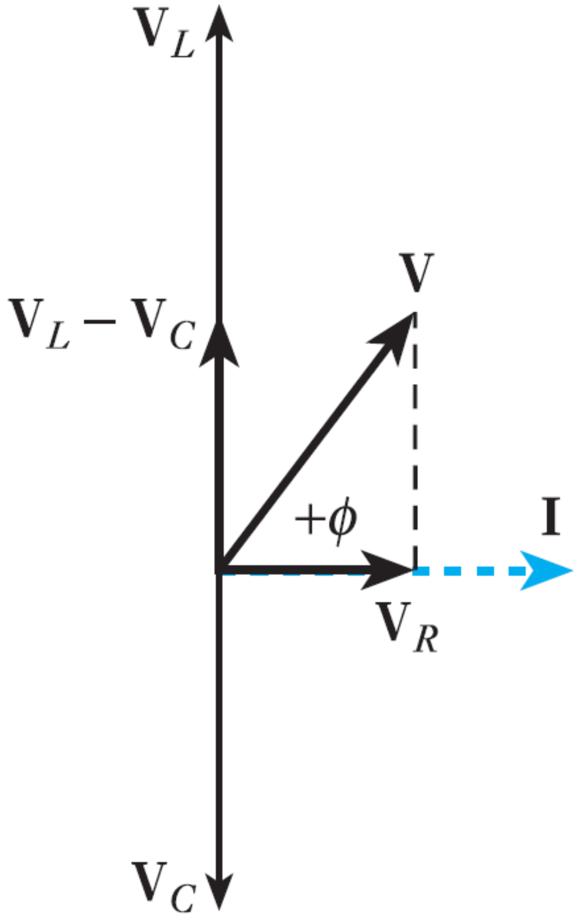
Voltages in A Series RLC Circuit



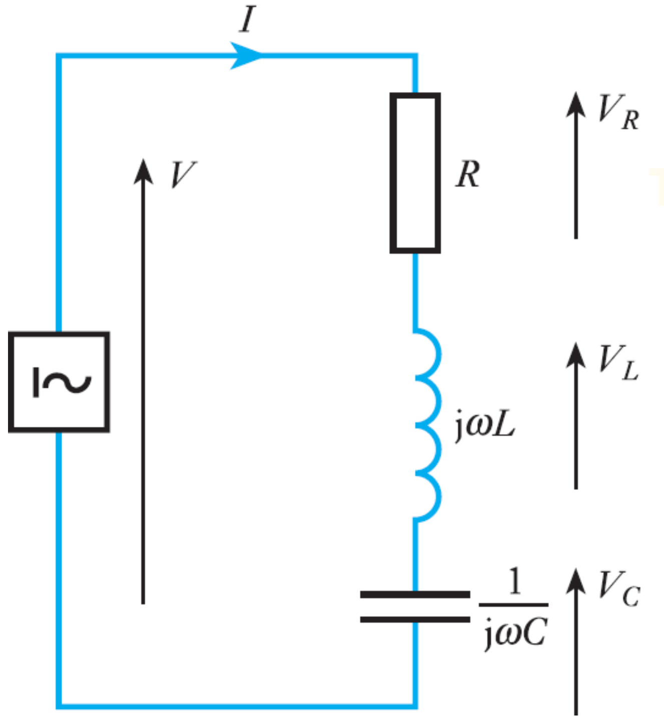
(a) $f < f_r$
Capacitive,
I leads *V*



(b) $f = f_r$
Resistive,
V and *I* in phase



(c) $f > f_r$
Inductive,
I lags *V*



Voltages Across RLC Elements at Resonance

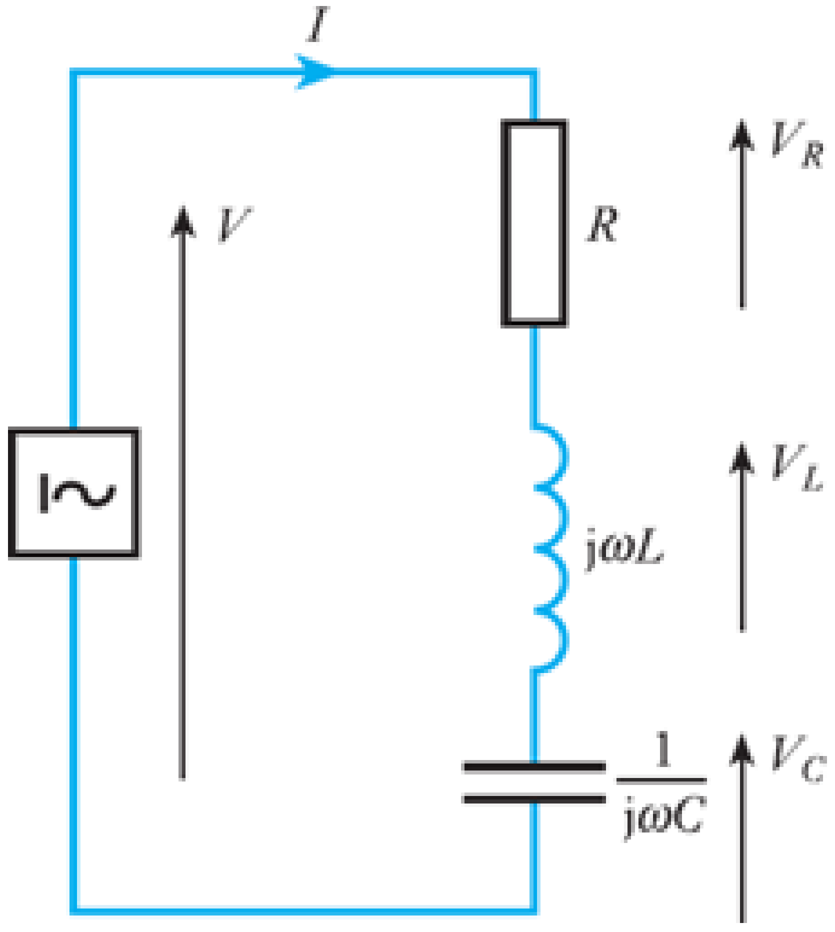
The voltage across resistor at f_r is,

$$V_R = I_R \times R = I_m \times R = \frac{V}{R} \times R \Rightarrow V_R = V$$

The voltage across inductor at f_r is,

$$|V_L| = X_L \times I_L = \omega_r L \times I_m = \omega_r L \times \frac{V}{R} = \frac{\omega_r L}{R} V = QV$$

$$\Rightarrow |V_L| = QV$$



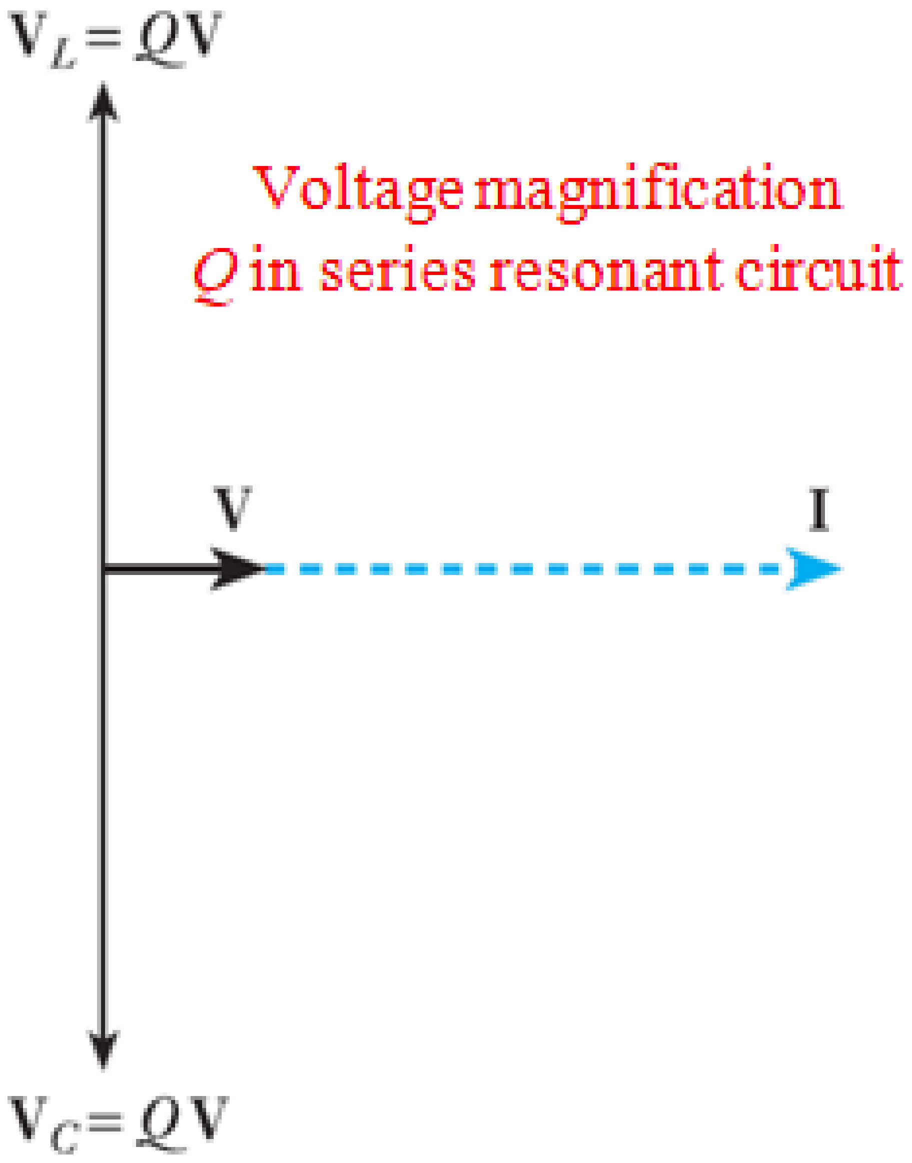
The voltage across capacitor at f_r is,

$$|V_C| = X_C \times I_C = \frac{1}{\omega_r C} \times I_m = \frac{1}{\omega_r C} \times \frac{V}{R} = \frac{1}{\omega_r CR} V = QV \Rightarrow |V_C| = QV$$

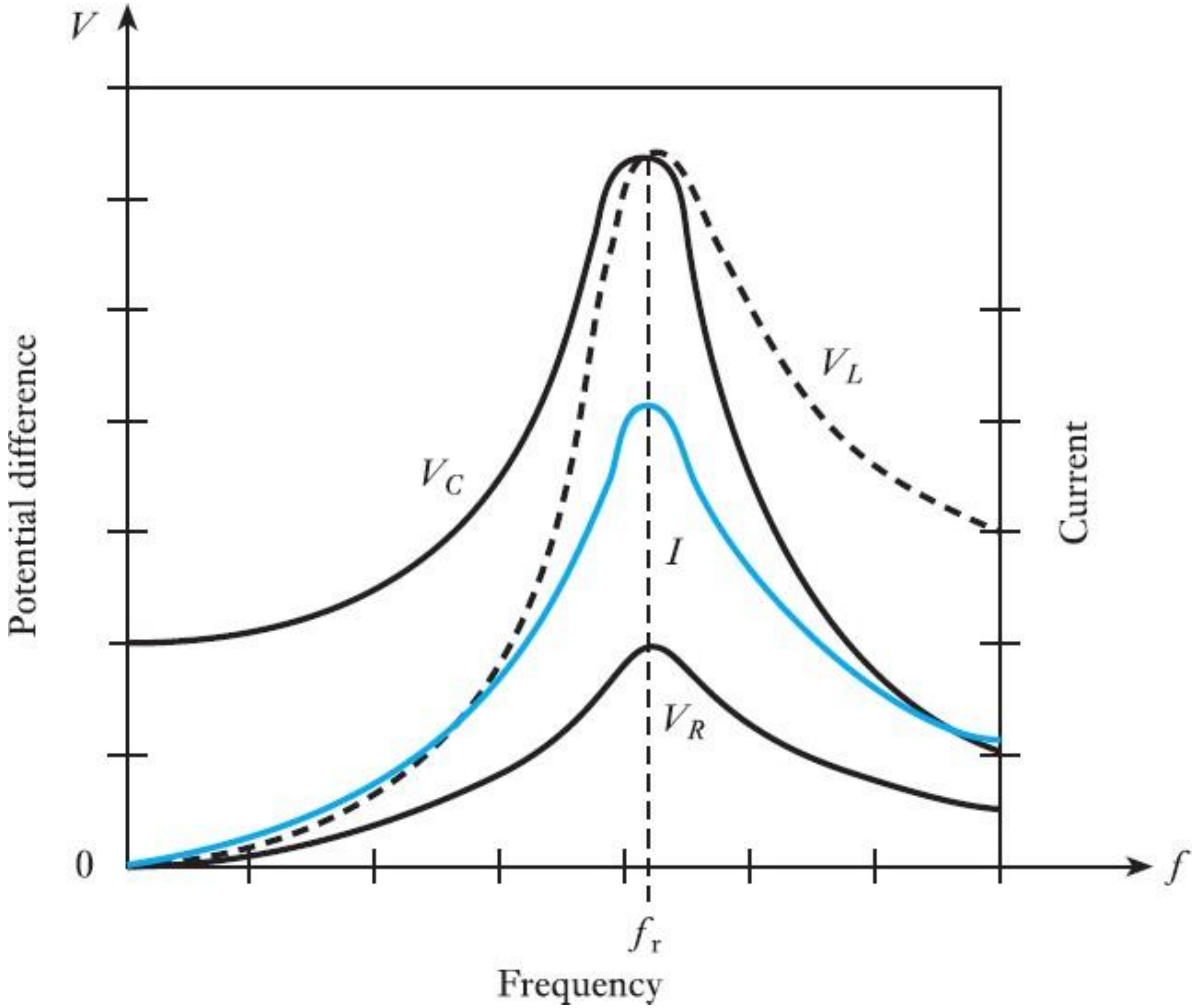
Voltages Across RLC Elements at Resonance

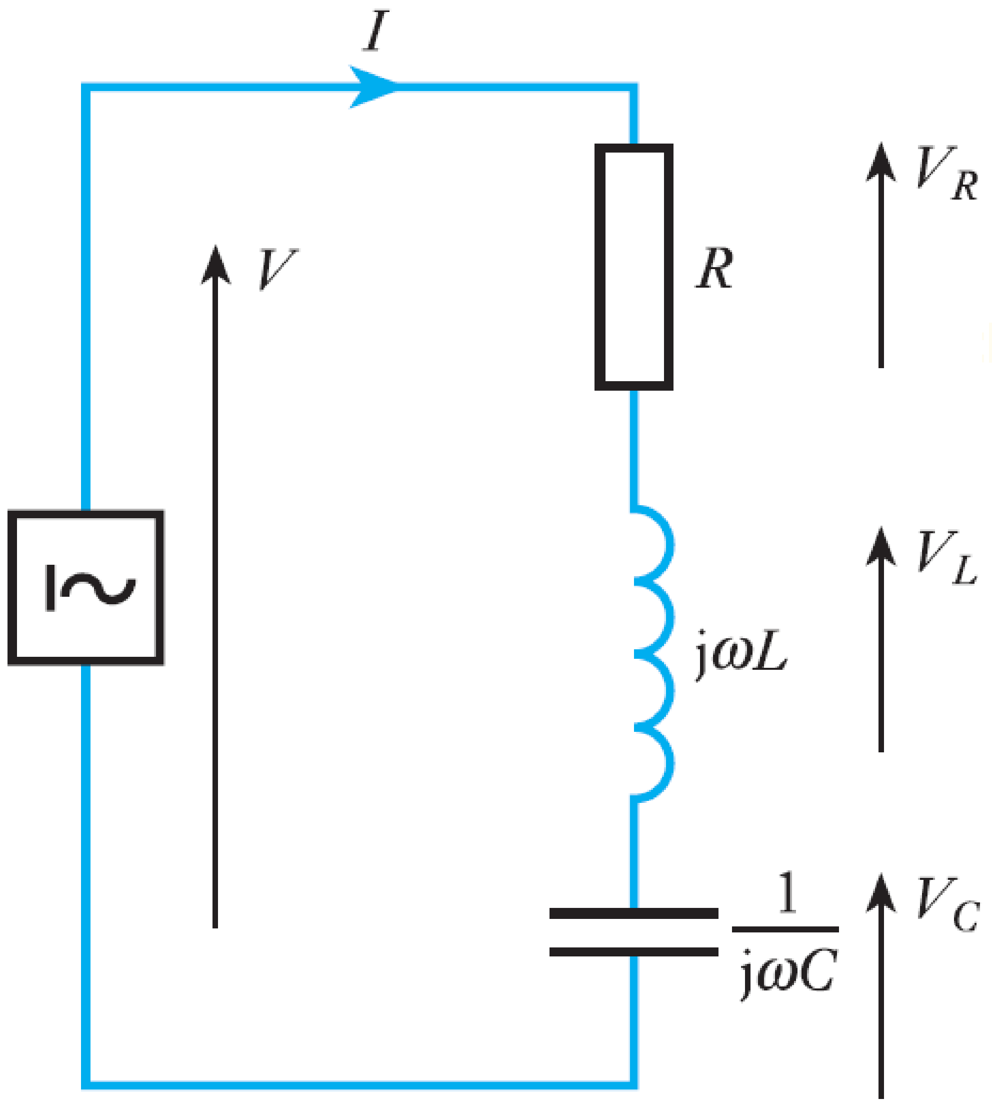
- Q is termed as Q factor or voltage magnification, because V_C or V_L equals Q multiplied by the source voltage V.
- In a series RLC circuit, values of V_L and V_C can actually be very large at resonance and can lead to component damage if not recognized and subject to careful design.

$$Q = \frac{\omega L}{R} = \frac{1}{\omega C R} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$



Voltages Across RLC Elements at Resonance





Bandwidth and Half Power Frequencies

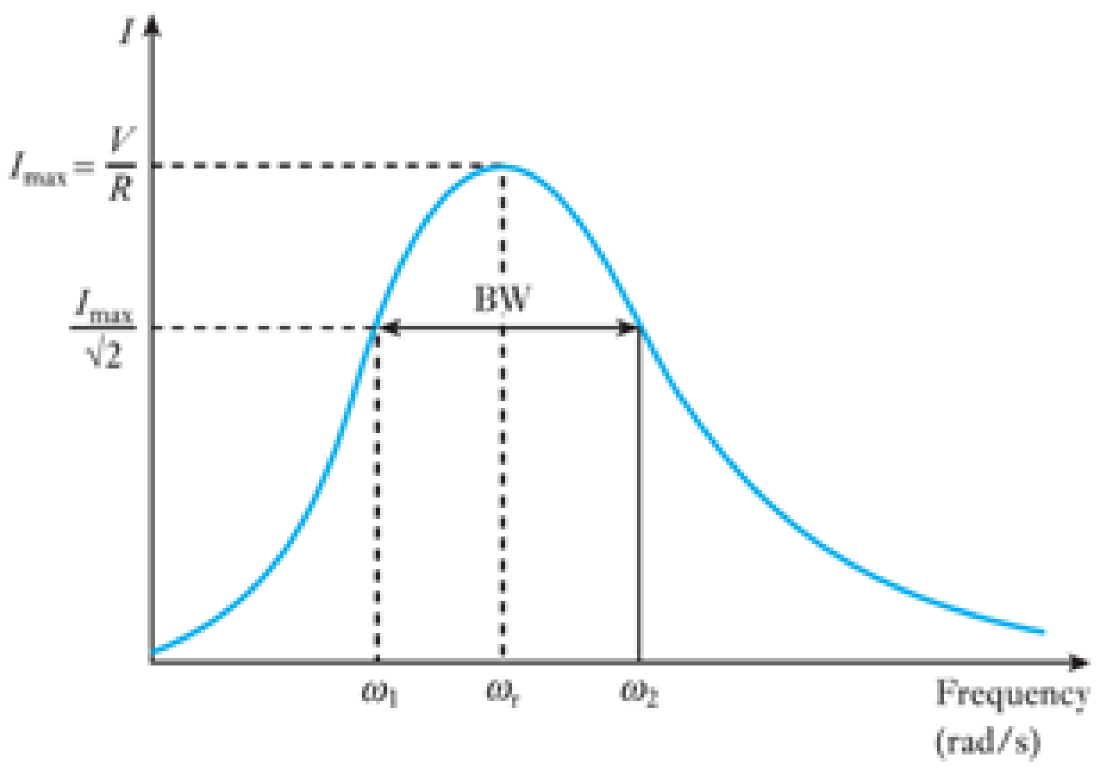
- In a series *RLC* circuit, at resonance, maximum power is drawn. i.e.,

$$P_r = I_{max}^2 \times R; \text{ where } I_{max} = \frac{V}{R} \text{ at resonance}$$

- Bandwidth represents the range of frequencies for which the power level in the signal is at least half of the maximum power.

$$\frac{P_r}{2} = \frac{I_{max}^2 \times R}{2} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 \times R$$

- The bandwidth of a circuit is also defined as the frequency range between the half-power points when $I = I_{max}/\sqrt{2}$.



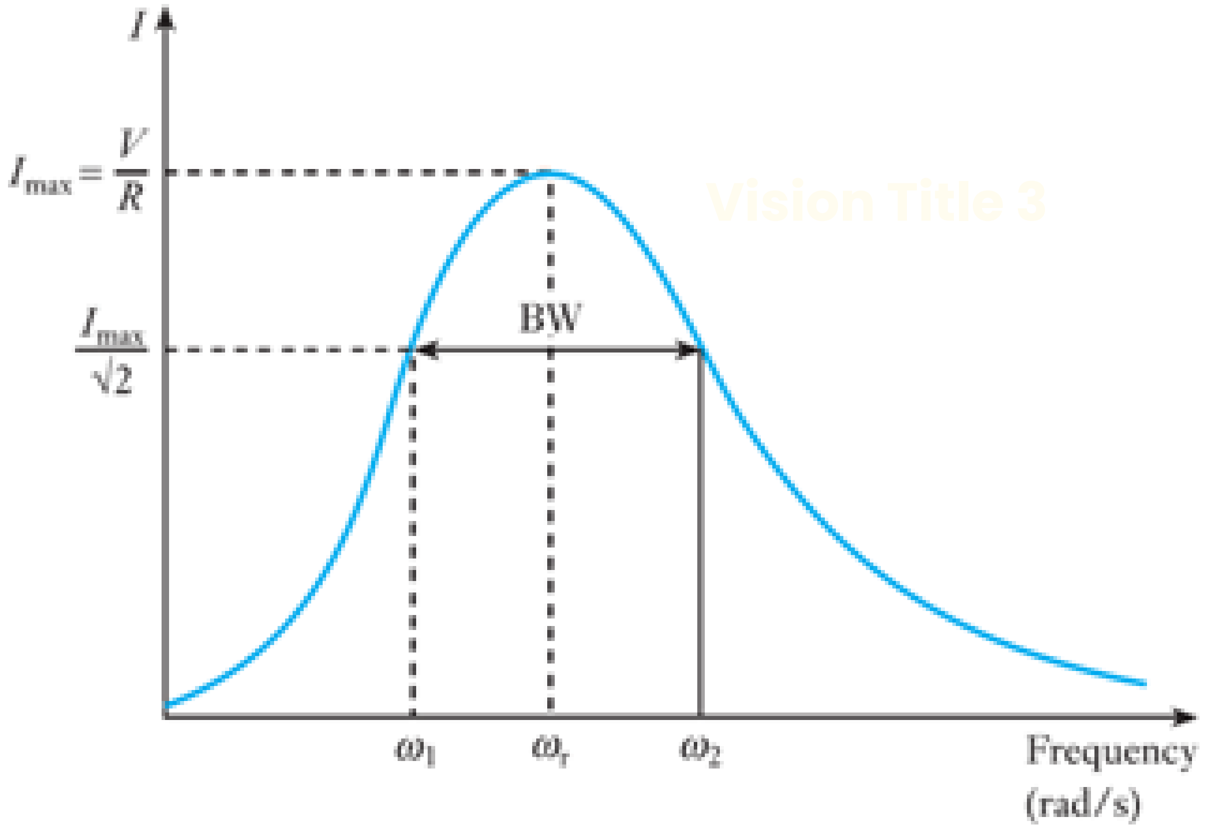
Bandwidth and Half Power Frequencies

- Thus, the condition for half-power is given when

$$|I| = \frac{I_{max}}{\sqrt{2}} = \frac{V}{R\sqrt{2}}$$

- The vertical lines either side of $|I|$ indicate that only the magnitude of the current is under consideration – but the phase angle will not be neglected.
- The impedance corresponding to half power-points including phase angle is

$$Z(\omega_{1,2}) = R\ \sqrt{2}\angle \pm 45^{\circ}$$



The resonance peak, bandwidth and half-power frequencies

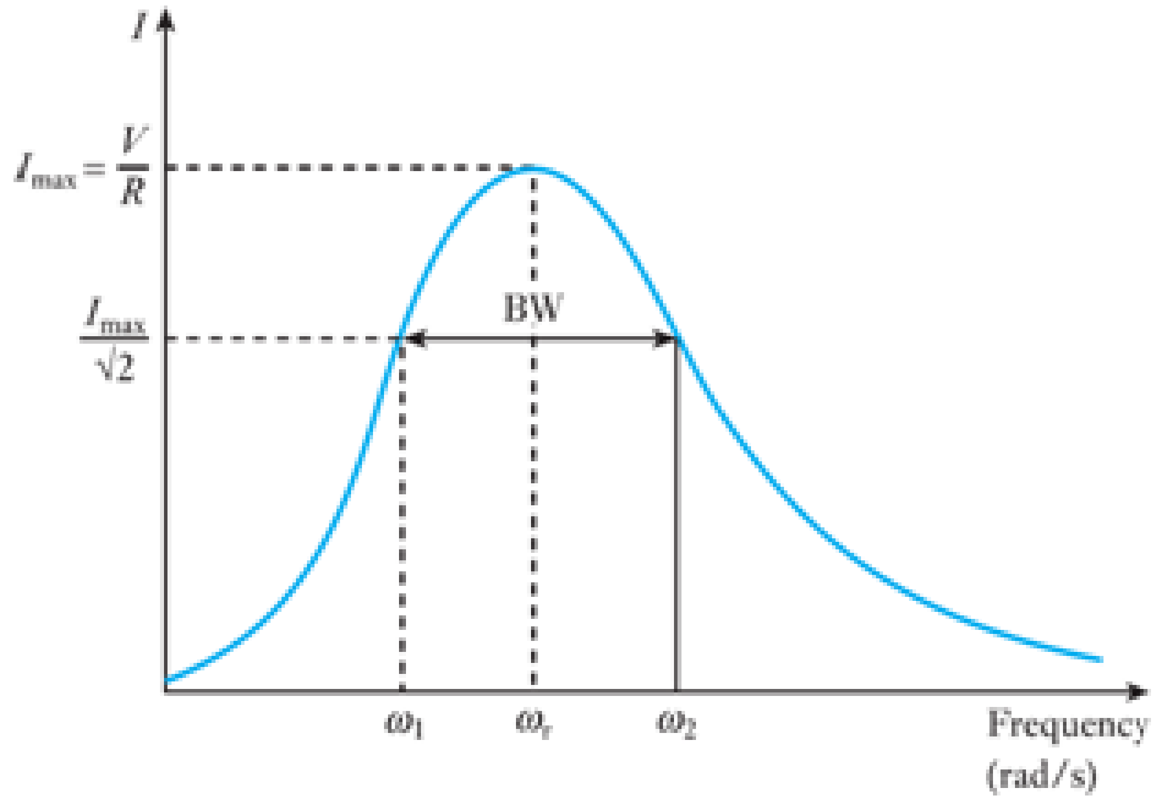
Bandwidth and Half Power Frequencies

- The impedance in the complex form

$$Z(\omega_{1,2}) = R(1 \pm j1)$$

- Thus for half power,

$$I = \frac{V}{R(1 \pm j1)} \quad \text{and} \quad Z = R(1 \pm j1)$$

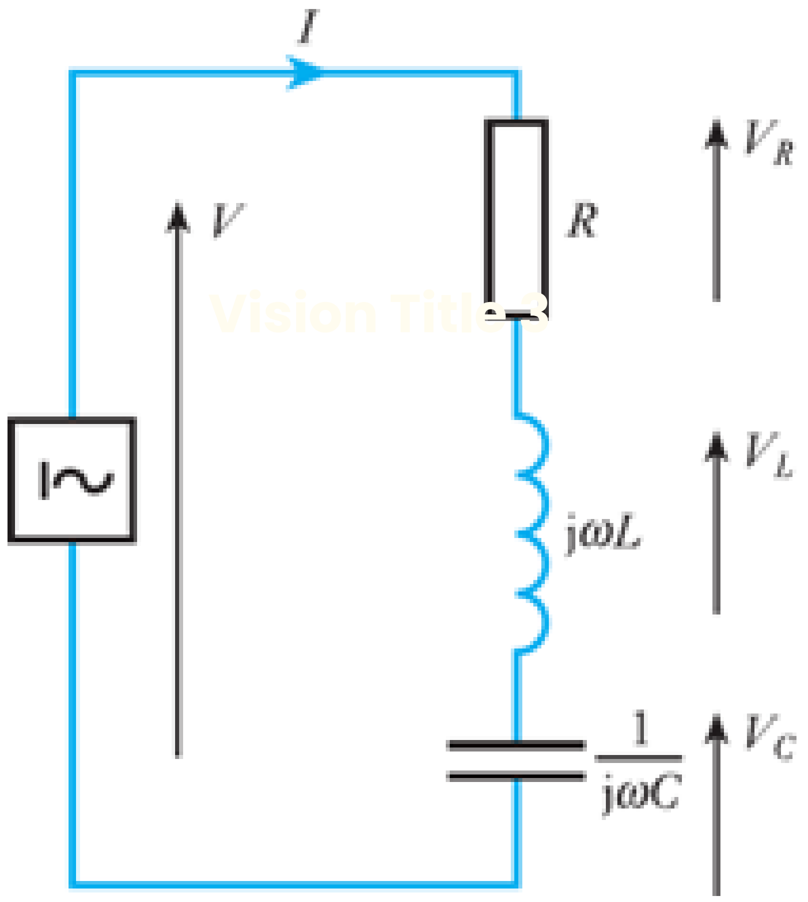


- At the half-power points, the phase angle of the current is 45°. Below the resonant frequency, at ω_1, the circuit is capacitive and $Z(\omega_1) = R(1 - j1)$.
- Above the resonant frequency, at ω_2, the circuit is inductive and $Z(\omega_2) = R(1 + j1)$.

Bandwidth and Half Power Frequencies

- Now, the circuit impedance is given by,

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R\left(1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)\right)$$



- At half power points, $Z = R(1 \pm j1)$
- By comparison of above two equations, resulting in

$$\frac{\omega L}{R} - \frac{1}{\omega CR} = \pm 1$$

- As we know,

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR}$$

Bandwidth and Half Power Frequencies

- Now, by multiplying and dividing with ω_r :

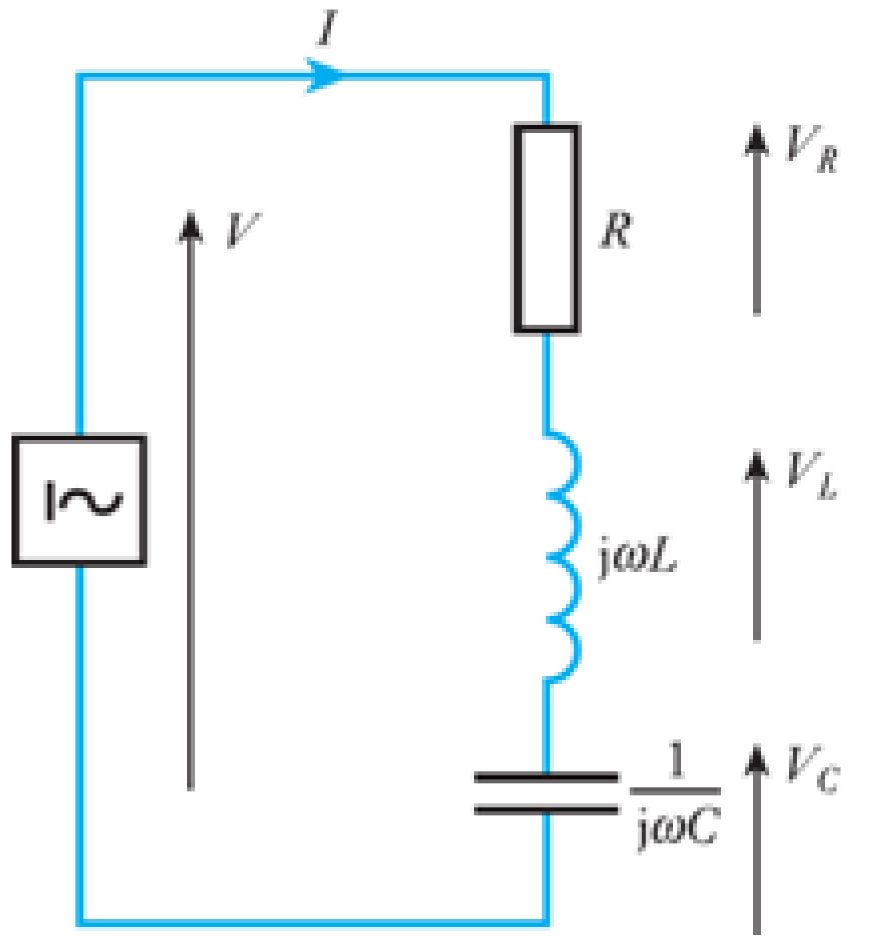
$$\frac{\omega L}{R}\frac{\omega_r}{\omega_r} - \frac{1}{\omega CR}\frac{\omega_r}{\omega_r} = \pm 1 \Rightarrow \frac{\omega}{\omega_r}Q - \frac{\omega_r}{\omega}Q = \pm 1 \Rightarrow Q\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right) = \pm 1$$

- For ω_2 :

$$Q\left(\frac{\omega_2}{\omega_r} - \frac{\omega_r}{\omega_2}\right) = 1$$

- For ω_1 :

$$Q\left(\frac{\omega_1}{\omega_r} - \frac{\omega_r}{\omega_1}\right) = -1$$

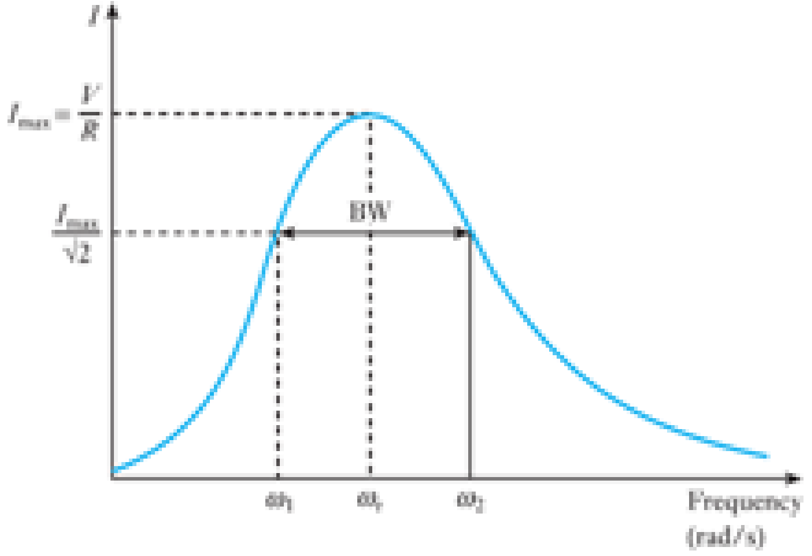


Bandwidth and Half Power Frequencies

- The half-power frequencies ω_2 and ω_1 are obtained as,

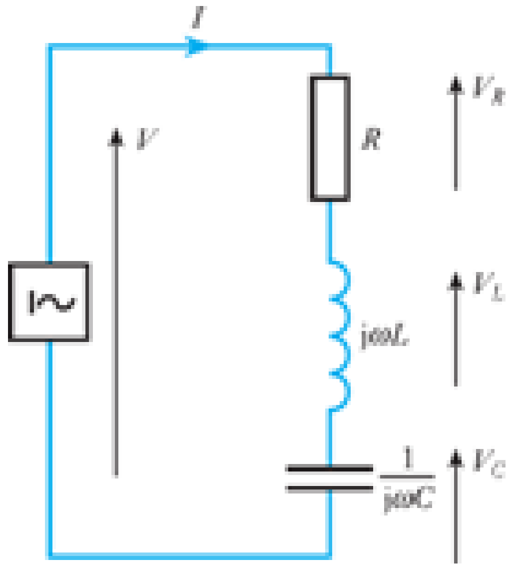
$$\omega_2 = \frac{\omega_r}{2Q} + \omega_r\sqrt{1 + \frac{1}{4Q^2}}$$

$$\omega_1 = \frac{-\omega_r}{2Q} + \omega_r\sqrt{1 + \frac{1}{4Q^2}}$$



- The bandwidth is obtained as:

$$BW = \omega_2 - \omega_1 = \frac{\omega_r}{Q}$$ i.e. $$\text{Bandwidth} = \frac{\text{Resonant frequency}}{Q\ \text{factor}}$$



- Resonant frequency in terms of ω_2 and ω_1, is expressed as:

$$\omega_r = \sqrt{\omega_1 \omega_2}$$

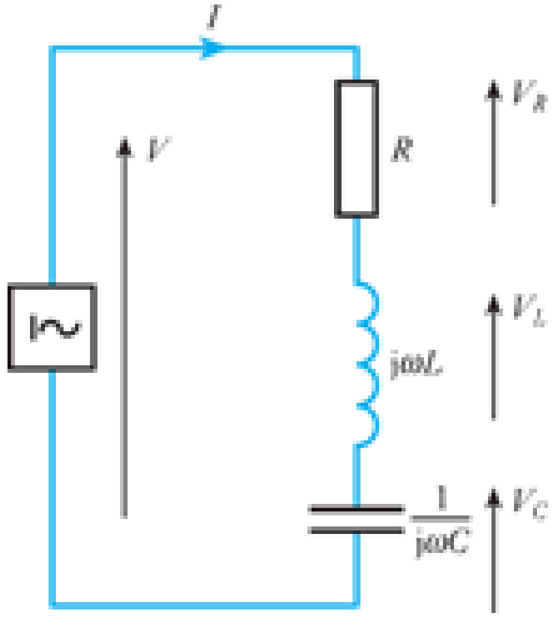
Bandwidth and Half Power Frequencies

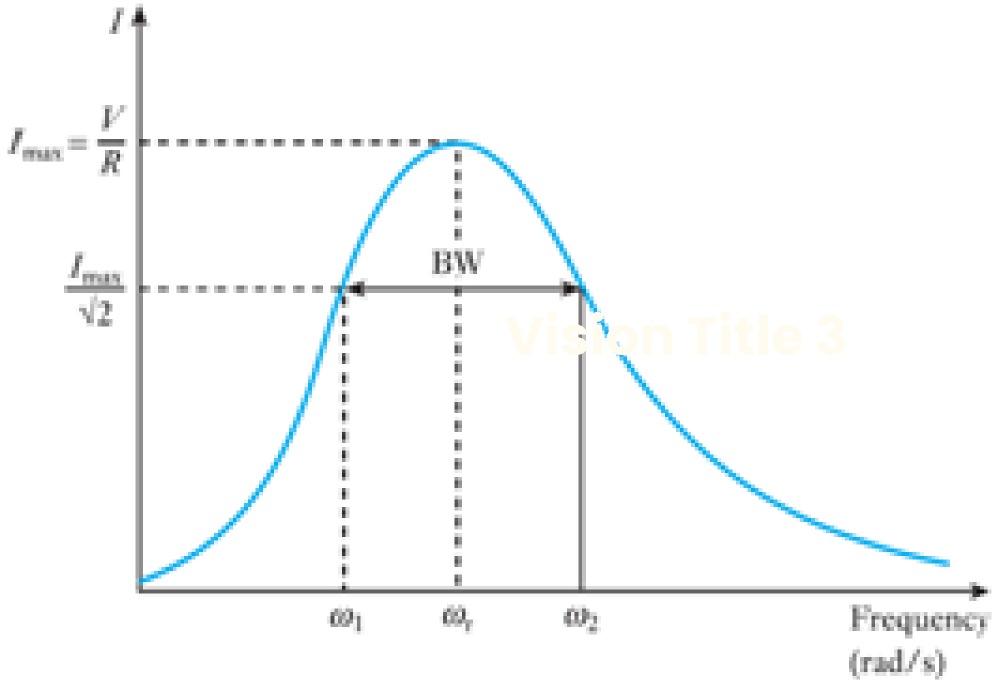
The bandwidth is also expressed as:

$$\omega_2 - \omega_1 = \frac{\omega_r}{Q} \Rightarrow \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/s}$$

(or)

$$f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$





For $Q >> 1$,

$$\omega_1 \approx \omega_r - \frac{BW}{2} \Rightarrow \omega_1 = \omega_r - \frac{BW}{2} \Rightarrow \omega_1 = \omega_r - \frac{R}{2L} \text{ rad/s}$$

$$\omega_2 \approx \omega_r + \frac{BW}{2} \Rightarrow \omega_2 = \omega_r + \frac{BW}{2} \Rightarrow \omega_2 = \omega_r + \frac{R}{2L} \text{ rad/s}$$