SNS COLLEGE OF TECHNOLOGY

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Department of Biomedical Engineering

Course Name: 23BMT201 & Circuit Analysis

I Year : II Semester

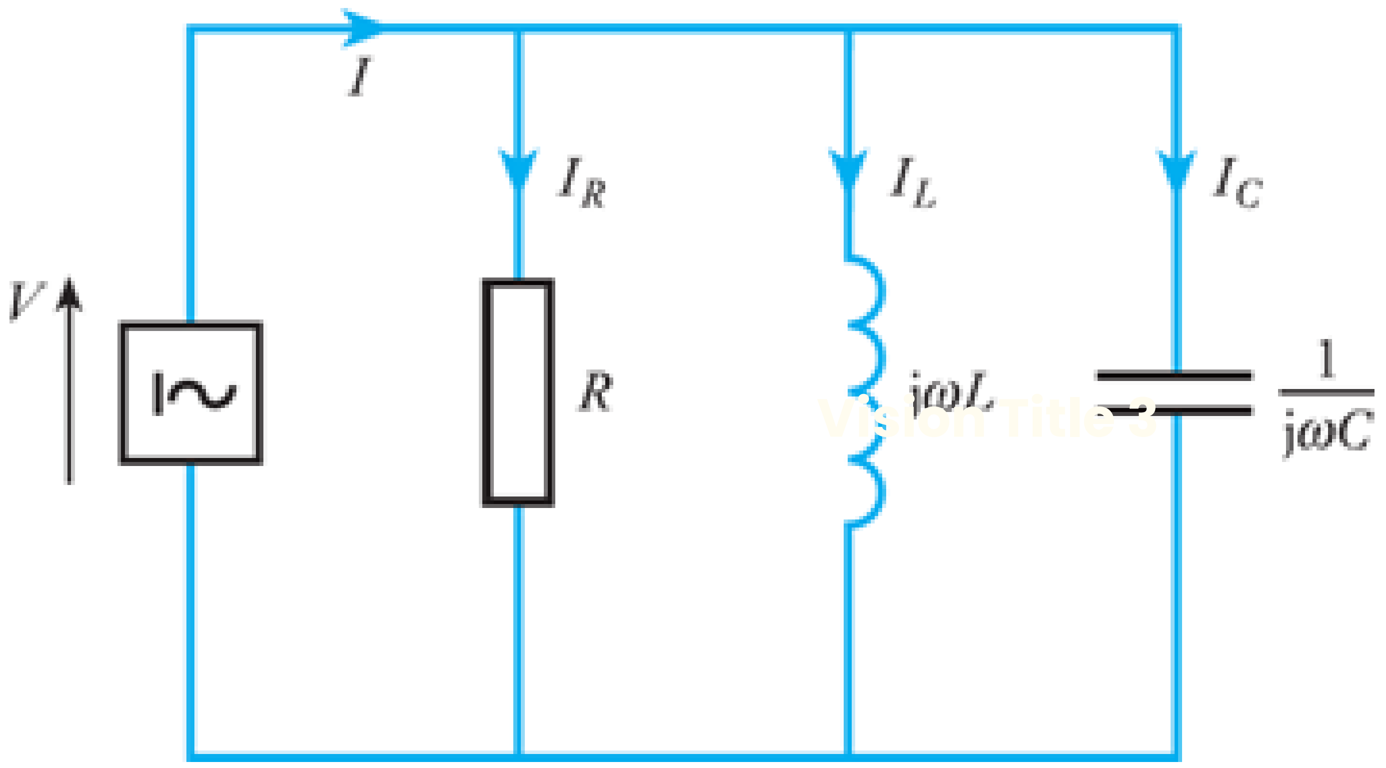
Unit V –RESONANCE CIRCUITS & COUPLED CIRCUITS

Topic : Parallel Resonance

Resonance in Parallel RLC Circuit

- The supply voltage: $V = IZ$ where Z is the net impedance of the three parallel branches.
- In parallel circuits, it is simpler to consider the total admittance Y of the three branches. Thus,

$$V = IZ = \frac{I}{Y}$$



where

$$Y = G + \frac{1}{j\omega L} + j\omega C = G - \frac{j}{\omega L} + j\omega C = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

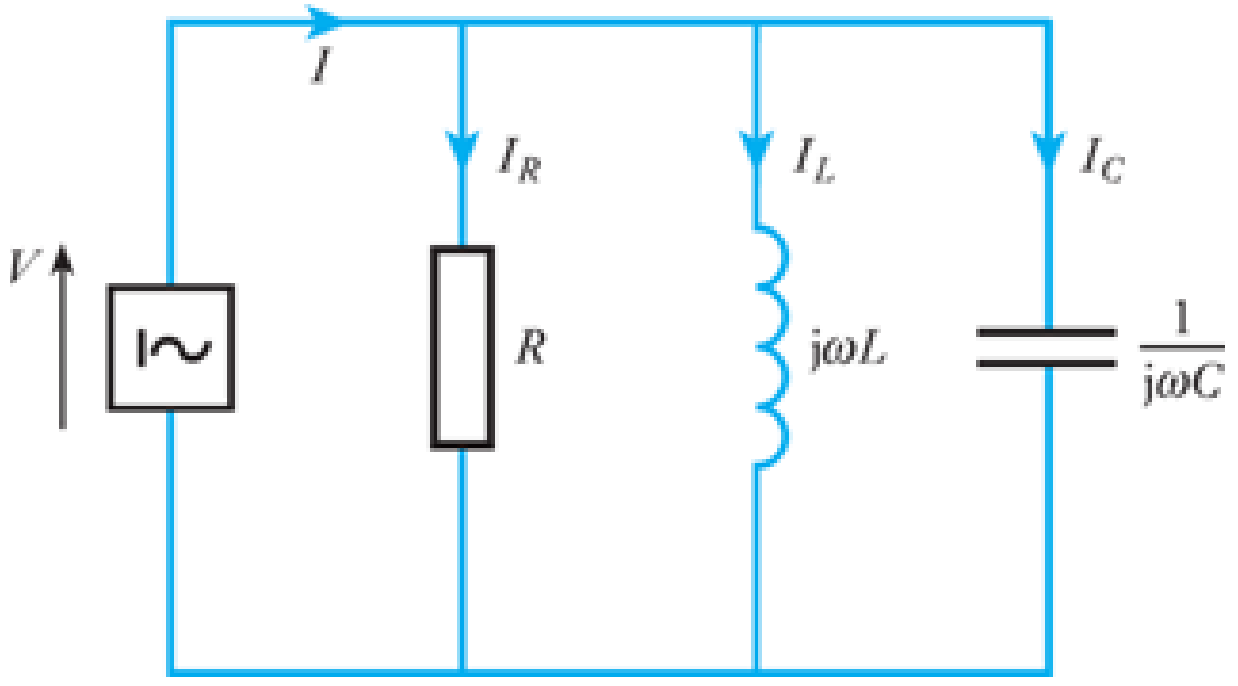
Resonance in Parallel RLC Circuit

- At resonance ($\omega = \omega_r$), the net susceptance is zero.

i.e. $$\left(\omega C - \frac{1}{\omega L}\right) = 0$$

- Therefore, the resonant frequency (ω_r) :

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/s}$$



- At the resonant frequency, $Y = G = 1/R$, the conductance of the parallel resistance, and $I = VG$.

Current through Resistance

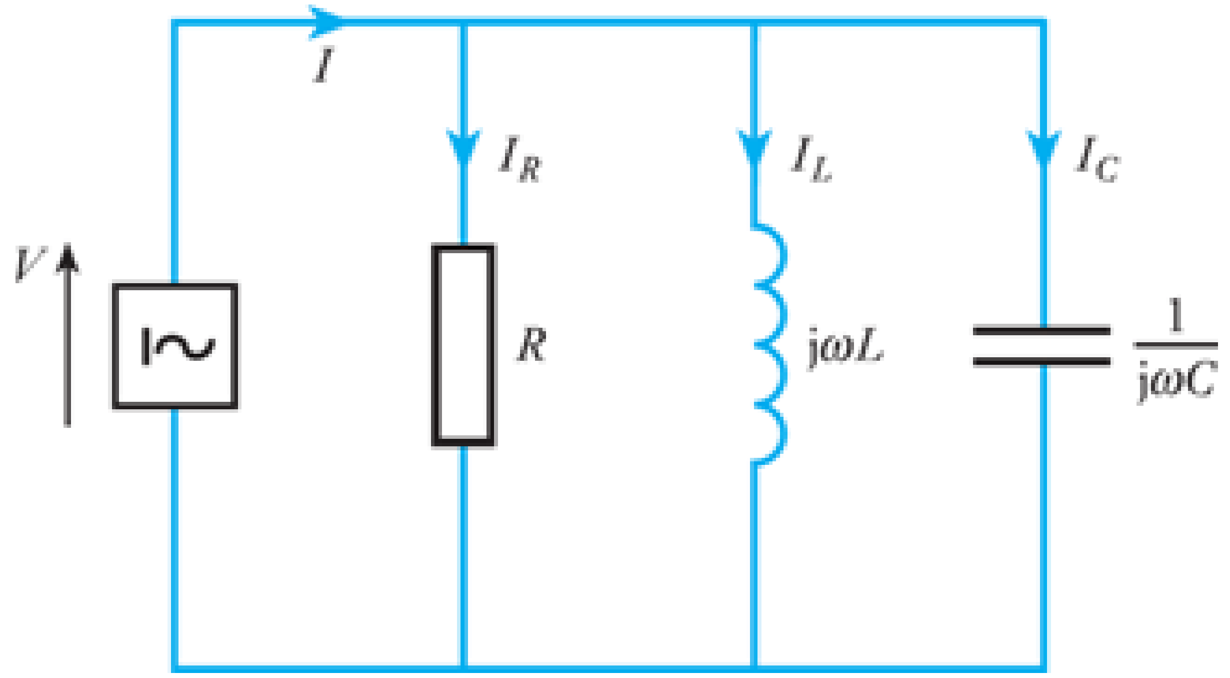
- The supply voltage magnitude:

$$|V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

- At resonance, $\omega = \omega_r$,

$$|V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + (0)^2}} \Rightarrow |V| = |I| \times R$$

- Current through the resistance at ω_r: $I_R = \frac{V_R}{R} = \frac{V}{R} = \frac{I \times R}{R} \Rightarrow I_R = I$



The three-branch
parallel resonant circuit

Current Magnification

- Magnitude of current through inductor at ω_r:

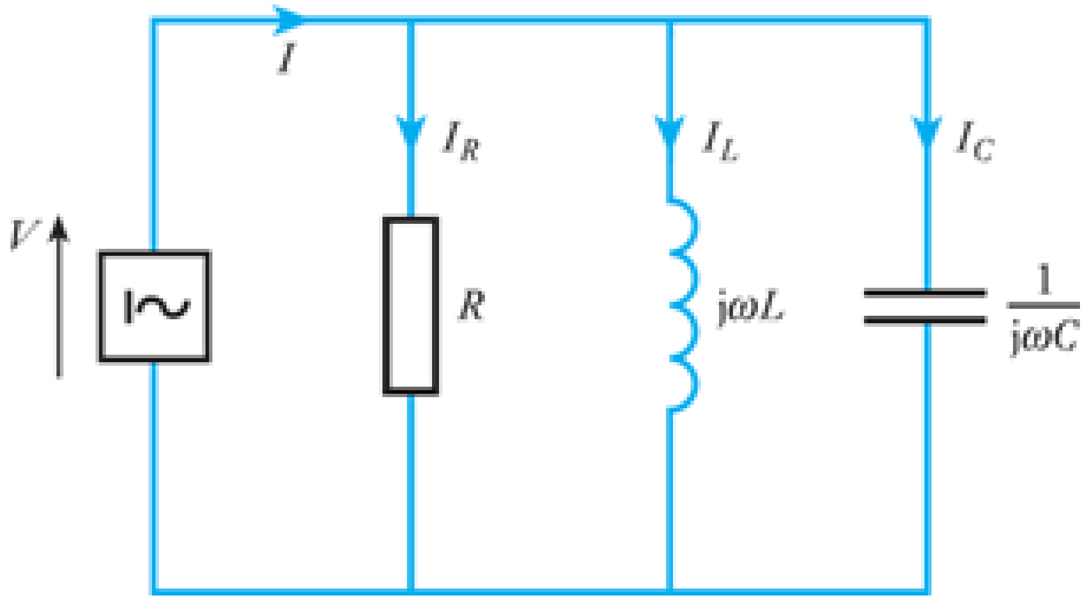
$$|I_L| = \frac{V}{X_L} = \frac{I \times R}{\omega_r L} = \left(\frac{R}{\omega_r L}\right) \times I = Q \times I$$

- Magnitude of current through capacitor at ω_r:

$$|I_C| = \frac{V}{X_C} = \frac{I \times R}{1/\omega_r C} = (\omega_r CR) \times I = Q \times I$$

where Q is the current magnification i.e.,

$$Q = \left(\frac{R}{\omega_r L}\right) = (\omega_r CR)$$



The three-branch parallel resonant circuit

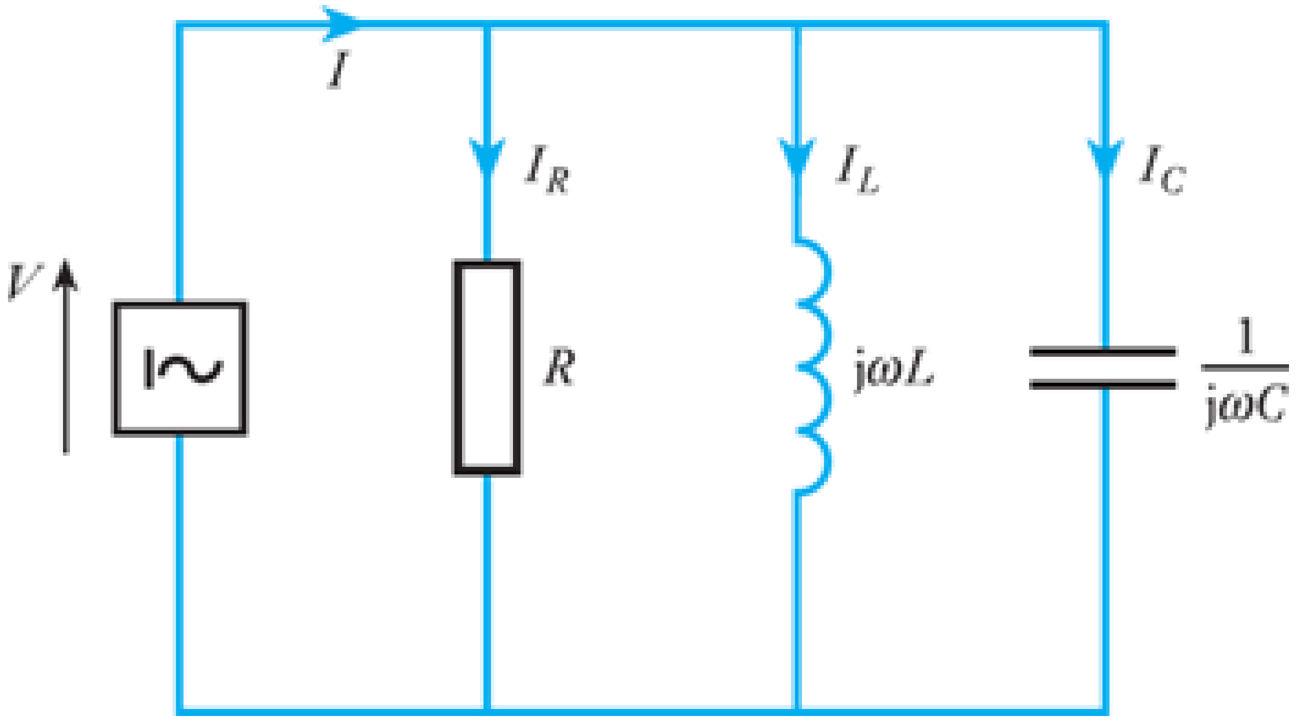
Current Magnification

Current magnification Q is also expressed in terms of inductive or capacitive susceptance (B), inductive or capacitive reactance (X) and conductance (G) :

$$Q = \left(\frac{1}{\omega_r L G}\right) = \left(\frac{\omega_r C}{G}\right) = \left(\frac{B}{G}\right) = \left(\frac{R}{X}\right)$$

By substituting $\omega_r = 1/\sqrt{(LC)}$ in Q :

$$Q = \frac{1}{G}\sqrt{\frac{C}{L}} = R\sqrt{\frac{C}{L}}$$



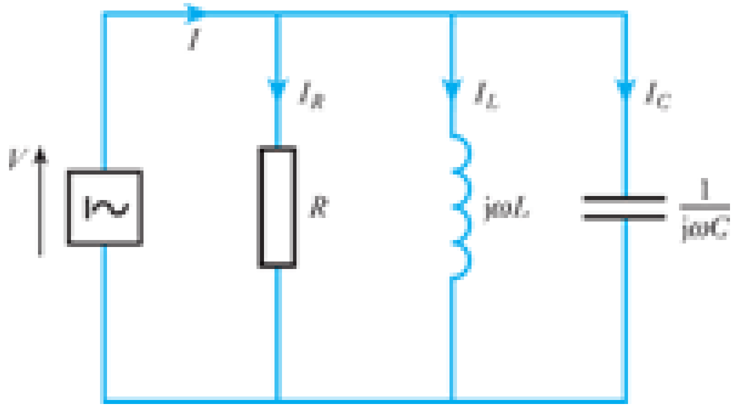
The three-branch parallel resonant circuit

Bandwidth and Half Power Frequencies

The parallel *RLC* circuit is the dual of the series *RLC* circuit. Therefore, by replacing *R*, *L*, and *C* in the expressions for the series circuit with 1/*R*, *C*, and *L* respectively, we obtain for the parallel circuit, the $Y_{min}/2^{1/2}$ frequencies:

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$



- Bandwidth: $BW = \omega_2 - \omega_1 = \frac{1}{RC}$

- Relation between BW and *Q*: $Q = \frac{\omega_0}{BW} = \omega_0 RC = \frac{R}{\omega_0 L}$

Bandwidth and Half Power Frequencies

The half-power frequencies in terms of quality factor:

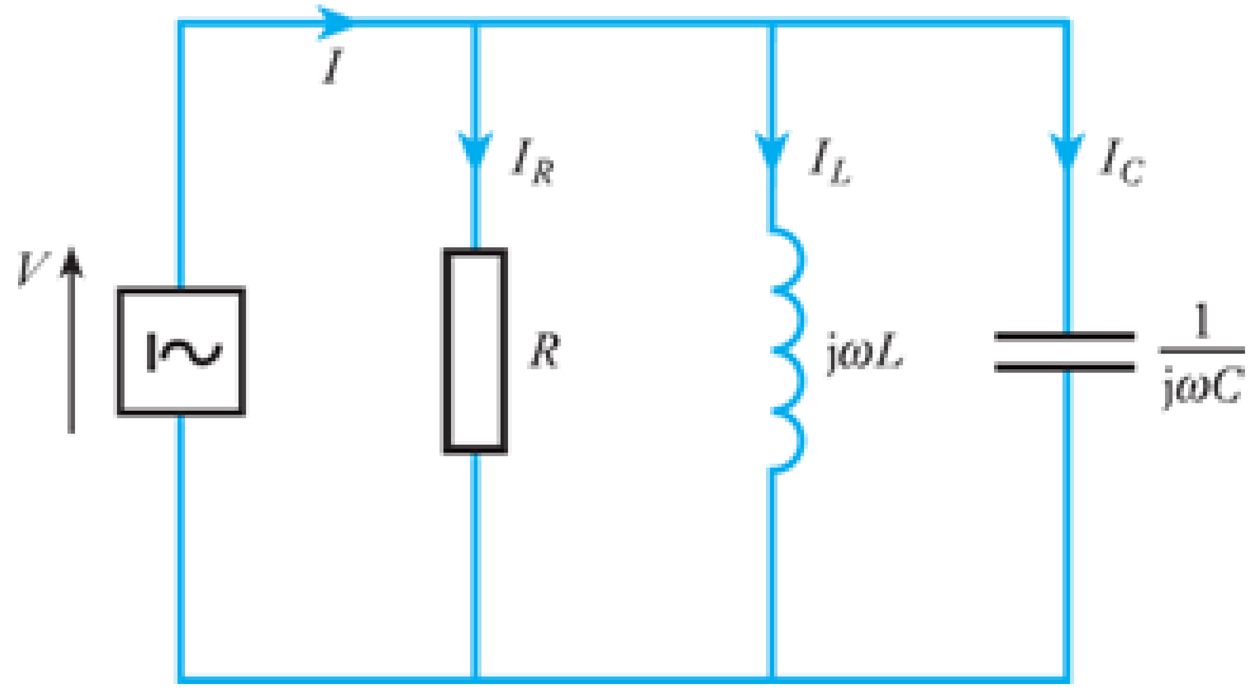
$$\omega_1 = \omega_r \sqrt{1+\left(\frac{1}{2Q}\right)^2} - \frac{\omega_r}{2Q}$$

$$\omega_2 = \omega_r \sqrt{1+\left(\frac{1}{2Q}\right)^2} + \frac{\omega_r}{2Q}$$

For $Q >> 1$,

$$\omega_r - \omega_1 = \frac{BW}{2}$$

$$\omega_2 - \omega_r = \frac{BW}{2}$$



The three-branch parallel resonant circuit