# SNS COLLEGE OF TECHNOLOGY
*(An Autonomous Institution)*
*Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai*
*Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &amp;*
*Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)*
*COIMBATORE-641 035, TAMIL NADU*

**DEPARTMENT OF MATHEMATICS**
**Unit 5 - Multiple Integrals**

Unit -V

## Multiple Integrals

1. Evaluate $\int_0^1 \int_1^2 x(x+y)\,dy\,dx$.

**Sol**

$$\int_0^1 \int_1^2 x(x+y)\,dy\,dx = \int_0^1 \int_1^2 (x^2+xy)\,dy\,dx$$

$$= \int_0^1 \left[x^2 y + \frac{xy^2}{2}\right]_{y=1}^{y=2} dx$$

$$= \int_0^1 \left[(2x^2+2x) - \left(x^2+\frac{x}{2}\right)\right] dx$$

$$= \int_0^1 \left[2x^2+2x-x^2-\frac{x}{2}\right] dx = \int_0^1 \left[x^2+\frac{3}{2}x\right] dx$$

$$= \left[\frac{x^3}{3} + \frac{3}{2}\frac{x^2}{2}\right]_0^1 = \left(\frac{1}{3}+\frac{3}{4}\right) - (0+0)$$

$$= \frac{13}{2}$$

2. Evaluate $\int_1^a \int_2^b \frac{1}{xy}\,dx\,dy$.

**Sol**

Let $I = \int_1^a \int_2^b \frac{1}{xy}\,dx\,dy$

$$= \left[\int_1^a \frac{1}{y}dy\right]\left[\int_2^b \frac{1}{x}dx\right]$$

$$= [\log y]_1^a \, [\log x]_2^b$$

$$= [\log a - \log 1][\log b - \log 2]$$

$$= [\log a][\log \tfrac{b}{2}]$$

**formula :**
1. $\int \frac{1}{x}dx = \log x$
2. $\log 1 = 0$

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*DEPARTMENT OF MATHEMATICS*

3. Evaluate $\int_0^3 \int_0^2 e^{x+y} \, dy \, dx$

**Sol**

$\text{let } I = \int_0^3 \int_0^2 e^{x+y} \, dy \, dx$

$= \int_0^3 \int_0^2 e^x e^y \, dy \, dx$

$= \left[ \int_0^3 e^x \, dx \right] \left[ \int_0^2 e^y \, dy \right]$

$= [e^x]_0^3 \, [e^y]_0^2 = [e^3 - e^0][e^2 - e^0]$

$= [e^3 - 1][e^2 - 1]$

4. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dx \, dy$.

**Sol**

$\text{Let } I = \int_0^a \int_0^{\sqrt{a^2+x^2}} dx \, dy$.

$I = \int_0^a \int_0^{\sqrt{a^2+x^2}} dx \, dy = \int_0^a \int_0^{\sqrt{a^2-x^2}} dy \, dx \quad \text{(correct form)}$

$= \int_0^a [y]_{y=0}^{y=\sqrt{a^2-x^2}} \, dx$

$= \int_0^a \left[ \sqrt{a^2-x^2} - 0 \right] dx$

$= \int_0^a \sqrt{a^2-x^2} \, dx$

$= \left[ \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$

$= \left[ \left( 0 + \frac{a^2}{2} \sin^{-1} 1 \right) - (0 + 0) \right] = \frac{a^2}{2} \left( \frac{\pi}{2} \right)$

$= \frac{\pi a^2}{4}$

1. $\sin^{-1} 0 = 0$

2. $\sin^{-1} 1 = \pi/2$

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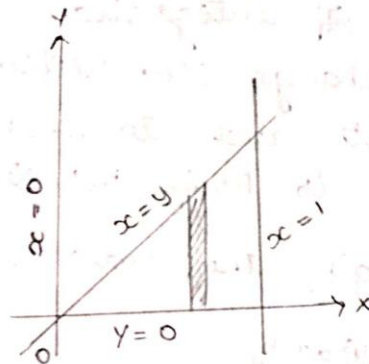*COIMBATORE-641 035, TAMIL NADU*

### DEPARTMENT OF MATHEMATICS

Sketch roughly the region of intergration

for $\int_0^1 \int_0^x f(x,y) \, dy \, dx$.

**Sol**

Gn: $\int_0^1 \int_0^x f(x,y) \, dy \, dx$.

$x$ varies from $x=0$ to $x=1$

$y$ varies from $y=0$ to $y=x$