



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



Approved by AICTE, New Delhi, Affiliated to Anna University, Chen

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU

DEPARTMENT OF MATHEMATICS

Area enclosed by plane curves:

1. Evaluate $\iint_R xy \, dx \, dy$, where R is the domain bounded by x axis, ordinate $x=2a$ and the curve $x^2=4ay$.

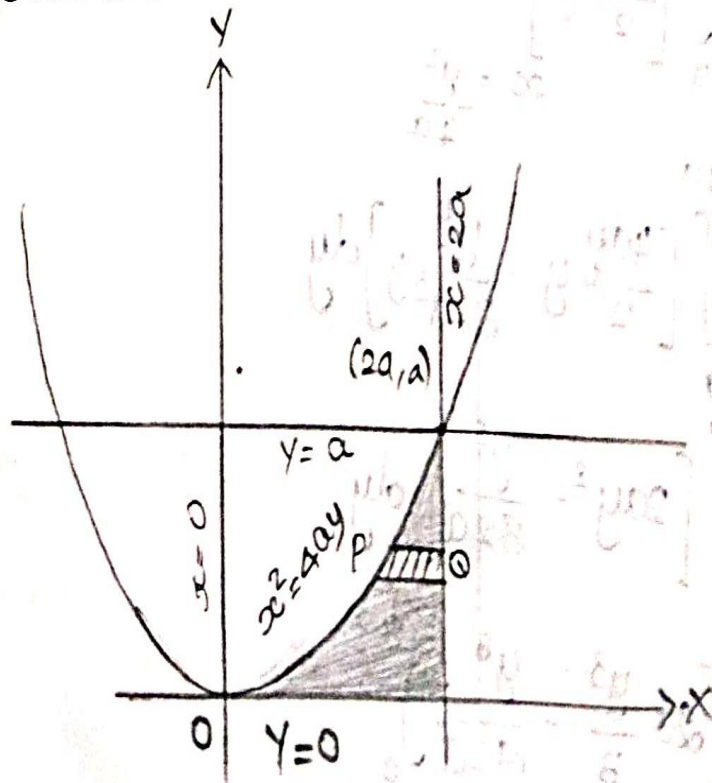
Sol

Ans x -axis $\Rightarrow y=0$ line

$$x = 2a$$

$$x^2 = 4ay$$

$$y = \frac{x^2}{4a}$$





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x	0	$2a$
y	0	a

Here, y limits varies from $y = 0$ to $y = a$
(Horizontal path).

x limits varies from $x = 2\sqrt{ay}$ to $x = 2a$
(Horizontal strip PQ).

\therefore The required area = $\int_0^a \int_{2\sqrt{ay}}^{2a} xy \, dx \, dy$.

$$= \int_0^a \left[y \frac{x^2}{2} \right]_{x=2\sqrt{ay}}^{x=2a} dy$$

$$= \int_0^a \left[\frac{4a^2y}{2} - \frac{4ay^2}{2} \right] dy = \int_0^a [2a^2y - 2ay^2] dy$$

$$= \left[2a^2 \frac{y^2}{2} - 2a \frac{y^3}{3} \right]_0^a = \left(a^4 - \frac{2a^4}{3} \right) - (0 - 0)$$

$$= \frac{1}{3} a^4 \text{ square units}$$



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DEPARTMENT OF MATHEMATICS

2. Using double integral, find the area bounded by $y=x$ and $y=x^2$

Sol

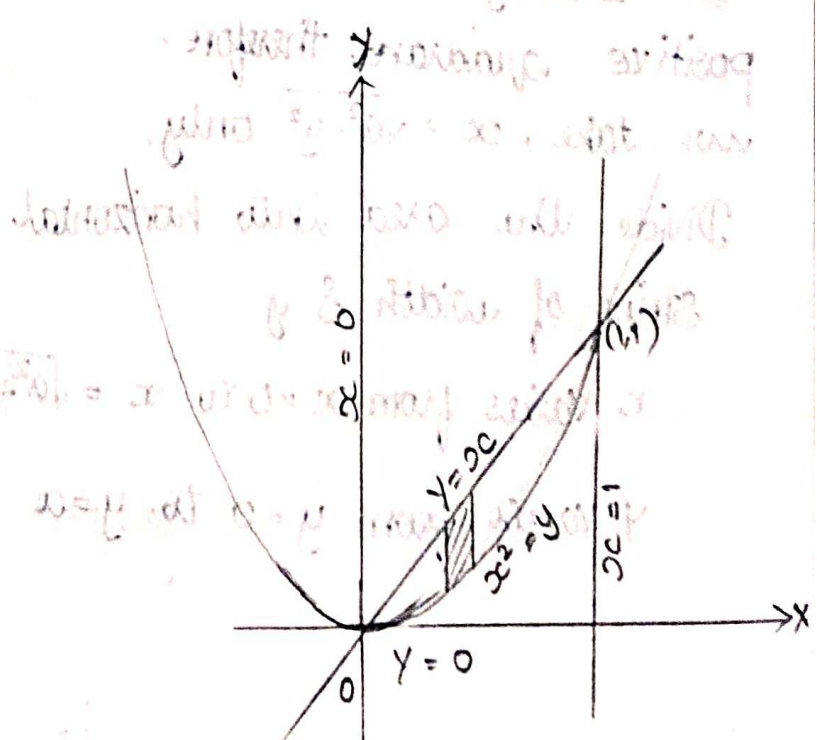
Ans: $y=x$ — ①

x	0	1	2	-1	-2
$y=x$	0	1	2	-1	-2

Ans.

$y=x^2$ — ②

x	0	1	2	-1	-2
$y=x^2$	0	1	4	1	4





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Therefore, the point of intersection of (1) and (2) is $(0,0)$ and $(1,1)$.

Divide the area into vertical strip of width δx .

x varies from $x=0$ to $x=1$ (vertical path)

y varies from $y=x^2$ to $y=x$ (vertical strip PQ)

$$\begin{aligned} \text{The required area} &= \int_0^1 \int_{x^2}^x dy dx \\ &= \int_0^1 [y]_{y=x^2}^{y=x} dx = \int_0^1 (x - x^2) dx \\ &= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \left(\frac{1}{2} - \frac{1}{3} \right) - (0 - 0) \\ &= \frac{3-2}{6} = \frac{1}{6} \text{ square unit.} \end{aligned}$$



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3. Evaluate: $\iint xy \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

Sol.

Ans:

$$x^2 + y^2 = a^2$$

$$x^2 = a^2 - y^2$$

$$x = \pm \sqrt{a^2 - y^2}$$

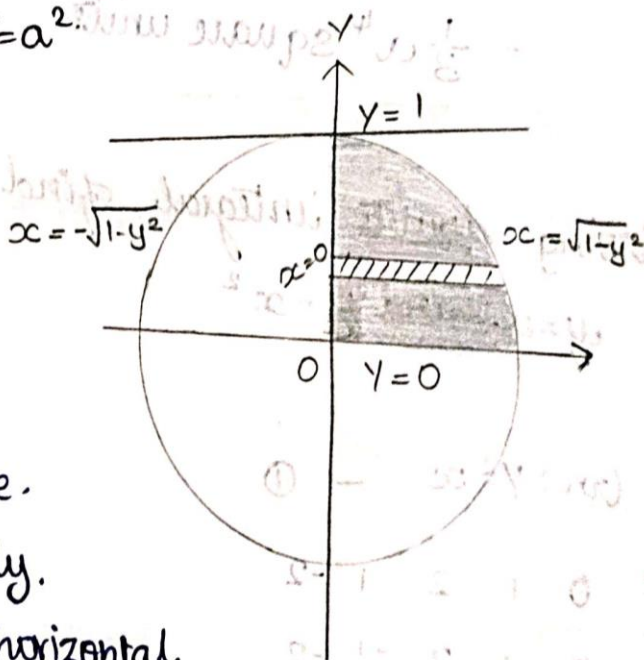
positive quadrant therefore,

we take, $x = \sqrt{a^2 - y^2}$ only.

Divide the area into horizontal strips of width δy

x varies from $x = 0$ to $x = \sqrt{a^2 - y^2}$

y varies from $y = 0$ to $y = a$





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$$\begin{aligned}\therefore \text{The required area} &= \int_0^a \int_0^{\sqrt{a^2-y^2}} xy \, dx \, dy \\ &= \int_0^a \left[y \frac{x^2}{2} \right]_{x=0}^{x=\sqrt{a^2-y^2}} dy \\ &= \int_0^a \left[\frac{y(a^2-y^2)}{2} - 0 \right] dy \\ &= \frac{1}{2} \int_0^a (a^2y - y^3) dy = \frac{1}{2} \left[\frac{a^2y^2}{2} - \frac{y^4}{4} \right]_0^a \\ &= \frac{1}{4} \left(a^2 - \frac{1}{2} \right) \\ A &= \frac{1}{8} (2a^2 - 1)\end{aligned}$$

note: put $a=1$, we get $A = \frac{1}{8}$, put $a=2$, we get $A = \frac{7}{8}$

H.W

4. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



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Sol

Area of ellipse = 4 x area of quadrant.

Divide the area into horizontal strips of width δy

x varies from $x=0$ to $x=\frac{a}{b}\sqrt{b^2-y^2}$

y varies from $y=0$ to $y=b$

\therefore The required area

$$= 4 \int_0^b \left[\frac{a}{b} \sqrt{b^2-y^2} \right] dy$$

$$= 4 \int_0^b \left[x \right]_0^{\frac{a}{b} \sqrt{b^2-y^2}} dy$$

$$= 4 \int_0^b \left[\frac{a}{b} \sqrt{b^2-y^2} - 0 \right] dy$$

$$= \frac{4a}{b} \int_0^b \sqrt{b^2-y^2} dy = \frac{4a}{b} \left[\frac{b^2}{2} \sin^{-1} \frac{y}{b} + \frac{y}{2} \sqrt{b^2-y^2} \right]_0^b$$

$$= \frac{4a}{b} \left[\frac{b^2}{2} \frac{\pi}{2} + 0 \right] - (0+0) = \frac{4a}{b} \frac{b^2}{2} \frac{\pi}{2}$$

$$= \pi ab \text{ square units}$$

