SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

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DEPARTMENT OF MATHEMATICS

Triple Integrals

1. Evaluate $\int_0^a\int_0^b\int_0^c xyz\,dz\,dy\,dx$

Sol

Let $I = \int_0^a\int_0^b\int_0^a xyz\,dz\,dy\,dx$.

$= \left[\int_0^a x\,dx\right]\left[\int_0^b y\,dx\right]\left[\int_0^c z\,dz\right]$.

$= \left[\frac{x^2}{2}\right]_0^a \left[\frac{y^2}{2}\right]_0^b \left[\frac{z^2}{2}\right]_0^c$.

$= \left[\frac{a^2}{2}-0\right]\left[\frac{b^2}{2}-0\right]\left[\frac{c^2}{2}-0\right] = \frac{(abc)^2}{8}.$

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2. Evaluate $\int_0^a\int_0^b\int_0^c e^{x+y+z}\,dz\,dy\,dx$.

Sol

Let $I = \int_0^a\int_0^b\int_0^c e^{x+y+z}\,dz\,dy\,dx$

$= \int_0^a\int_0^b\int_0^c e^x e^y e^z\,dz\,dy\,dx$

$= \left[\int_0^a e^x dx\right]\left[\int_0^b e^y dy\right]\left[\int_0^c e^z dz\right]$

$= \left[e^x\right]_0^a \left[e^y\right]_0^b \left[e^z\right]_0^c$

$= (e^a - e^0)(e^b - e^0)(e^c - e^0)$

$= (e^a - 1)(e^b - 1)(e^c - 1)$.

3. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x-y} e^z \, dx\, dy\, dz$.

Sol

Let $I = \int_0^1 \int_0^{1-x} \int_0^{x+y} e^z \, dx\, dy\, dz$.

$= \int_0^1 \int_0^{1-x} \int_0^{x+y} e^z \, dz\, dy\, dx$. (crt form)

$= \int_0^1 \int_0^{1-x} [e^z]_0^{x+y} \, dy\, dx$

$= \int_0^1 [e^{x+y} - y]_{y=0}^{y=1-x} \, dx$

$= \int_0^1 [(e-(1-x)) - (e^x - 0)] \, dx$

$= \int_0^1 (e - 1 + x - e^x) \, dx$

$= \left[ex - x + \frac{x^2}{2} - e^x\right]_0^1$

$= \left[(e - 1 + \frac{1}{2} - e) - (0 - 0 + 0 - 1)\right]$

$= -\frac{1}{2} + 1 = \frac{1}{2}$

4. Express the region $x \geq 0, y \geq 0, z \geq 0$, $x^2+y^2+z^2 \leq 1$ by triple integration.

Sol

For the given region z varies from 0 to $\sqrt{1-x^2-y^2}$

Y varies from 0 to $\sqrt{1-x^2}$

Z varies from 0 to 1

$\therefore I = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} dz\, dy\, dx$