Area of us Double Integral

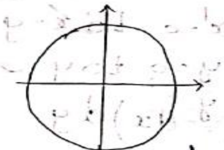
Formula

* If $\iint dx\,dy$ is given then draw the strip parallel to x-axis.

* $\iint dy\,dx$ is given then draw the strip parallel to y-axis

Example:-1

Evaluate $\iint xy\,dx\,dy$ where R is the first quadrant of the circle $x^2+y^2=a^2$ (x greater than or equal to zero)



x varies from $x=0$ to $x=\sqrt{a^2-y^2}$

y varies from $y=0$ to $y=a$.

$$=\int_0^a\int_0^{\sqrt{a^2-y^2}} xy\,dx\,dy$$

$$=\int_0^a y\left[\int_0^{\sqrt{a^2-y^2}} x\,dx\right]dy$$

$$=\int_0^a y\left[\frac{x^2}{2}\right]_0^{\sqrt{a^2-y^2}}dy$$

$$=\int_0^a y\left[\left(\frac{a^2-y^2}{2}\right)\right]dy$$

$$=\frac{1}{2}\left[a^2\int_0^a y\,dy-\int_0^a y^3\,dy\right]$$

$$=\frac{1}{2}\left[a^2\left(\frac{y^2}{2}\right)_0^a-\left(\frac{y^4}{4}\right)_0^a\right]$$

$$= \frac{1}{2}\left[\frac{a^4}{2} - \frac{a^4}{4}\right]$$

$$= \frac{1}{2}\left[\frac{4a^4 - 2a^4}{8}\right]$$

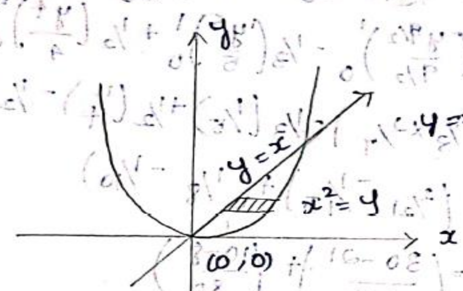
$$= \frac{1}{2}\left[\frac{2a^4}{8}\right]$$

$$= \frac{a^4}{8}$$

Example:- 2

Evalute $\iint_R xy(x+y)\,dx\,dy$ between the curve and a straight line such as $y = x^2$ and $y = x$

$\rightarrow$



$y = x^2$, $y = x$
or
$x^2 = y$, $x = y$

$y = x^2 \rightarrow (1)$ Take equ (2) $y = x$

$y = x \rightarrow (2)$ sub equ (1) in (2) $x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ (or) $x - 1 = 0$,

$x = 0$, $x = 1$

Sub $x = 0$ in equ (1) $y = 0$

Sub $x = 1$ in equ (2)

$y = 1$

x varies from $x = y$ to $x = \sqrt{y}$

y varies from $y = 0$ to $y = 1$

$$\int_0^1 \int_y^{\sqrt{y}} (x^2 y + x y^2)\,dx\,dy$$

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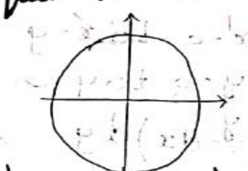
Example-1

Evaluate $\iint xy\,dx\,dy$ where R is the first quadrant of the circle $x^2+y^2=a^2$ (x greater than or equal to zero)



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y varies from $y=0$ to $y=a$.

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$$=\int_0^a y\left[\frac{x^2}{2}\right]_0^{\sqrt{a^2-y^2}} dy$$

$$=\int_0^a y\left[\left(\frac{a^2-y^2}{2}\right)\right]dy$$

$$=\frac{1}{2}\left[a^2\int_0^a y\,dy-\int_0^a y^3\,dy\right]$$

$$=\frac{1}{2}\left[a^2\left(\frac{y^2}{2}\right)_0^a-\left(\frac{y^4}{4}\right)_0^a\right]$$

$$= \int_0^1 y\left[\int_y^{\sqrt{y}} x^2 dx + y^2 \int_y^{\sqrt{y}} x\, dx\right] dy$$

$$= \int_0^1 \left[y\left[\frac{x^3}{3}\right]_y^{\sqrt{y}} + y^2\left[\frac{x^2}{2}\right]_y^{\sqrt{y}}\right] dy$$

$$= \int_0^1 \tfrac{1}{3}\left[y^2\sqrt{y} - y^4\right] + \tfrac{1}{2}\left[y^2 - y^4\right]\Big] dy$$

$$= \tfrac{1}{3}\int_0^1 y^{5/2} dy - \tfrac{1}{3}\int_0^1 y^4 dy + \tfrac{1}{2}\int_0^1 y^3 dy - \tfrac{1}{2}\int_0^1 y^4 dy$$

$$= \tfrac{1}{3}\left[\frac{y^{5/2+1}}{5/2+1}\right]_0^1 - \tfrac{1}{3}\left[\frac{y^5}{5}\right]_0^1 + \tfrac{1}{2}\left[\frac{y^4}{4}\right]_0^1 - \tfrac{1}{2}\left(\frac{y^5}{5}\right)_0^1$$

$$= \tfrac{1}{3}\left(\frac{y^{9/2}}{7/2}\right)_0^1 - \tfrac{1}{3}\left(\frac{y^5}{5}\right)_0^1 + \tfrac{1}{2}\left(\frac{y^4}{4}\right)_0^1 - \tfrac{1}{2}\left(\frac{y^5}{5}\right)_0^1$$

$$= \left(\tfrac{1}{3}\times\tfrac{2}{7}\right) - \tfrac{1}{3}\left(\tfrac{1}{5}\right) + \tfrac{1}{2}\left(\tfrac{1}{4}\right) - \tfrac{1}{2}\left(\tfrac{1}{5}\right)$$

$$= \left(\tfrac{2}{21} - \tfrac{1}{15}\right) + \left(\tfrac{1}{8} - \tfrac{1}{10}\right)$$

$$= \left(\frac{30-21}{315}\right) + \left(\frac{10-8}{80}\right)$$

$$= \frac{9}{315} + \frac{2}{80}$$

$$= \frac{3}{105} + \frac{1}{40}$$

$$= \frac{120+105}{4200}$$

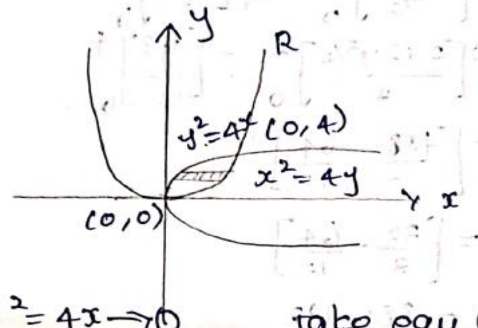
$$= \frac{225}{4200}$$

$$= \frac{3}{56}$$

Example :- 3

Find the area between the $\iint_R y^2 = 4x$ and $x^2 = 4y$

$\Longrightarrow$



$y^2 = 4x \rightarrow ①$

$x^2 = 4y \rightarrow ②$

Sub y in equ ①

$y^2 = 4x$

$\left(\frac{x^2}{4}\right)^2 = 4x$

$\frac{x^4}{16} = 4x$

$x^4 = 64x$

$x^3 = 4^3$

$x = 4$

take equ ②

$x^2 = 4y$

$\frac{x^2}{4} = y$

Sub $x = y$ in equ ②

$x^2 = 4y$

$(4)^2 = 4y$

$\frac{16}{4} = y$

$y = 4$

x varies from $x = \frac{y^2}{4}$ to $x = 2\sqrt{y}$

y varies from $y = 0$ to $y = 4$

$$\int_0^4 \left(\int_{y^2/4}^{2\sqrt{y}} dx \right) dy$$

$$= \int_0^4 [x]_{y^2/4}^{2\sqrt{y}} dy$$

$$= \int_0^4 [2\sqrt{y} - y^2/4]\, dy$$

$$= 2\int_0^4 y^{1/2} dy - \frac{1}{4}\int_0^4 y^2 dy$$

$$= 2\left[\frac{y^{1/2+1}}{1/2+1}\right]_0^4 - \frac{1}{4}\left[\frac{y^3}{3}\right]_0^4$$

$$= 2\left[\frac{y^{3/2}}{3/2}\right]_0^4 - \frac{1}{4}\left[\frac{y^3}{3}\right]_0^4$$

$$= 2\left[\frac{2\times 2y^{3/2}}{3}\right]_0^4 - \left[\frac{y^3}{12}\right]_0^4$$

$$= \left[\frac{4\times 8}{3} - \frac{64}{12}\right]$$

$$= \left[\frac{32}{3} - \frac{64}{12}\right]$$

$$= \frac{384-192}{36}$$

$$= \frac{192}{36}$$

$$= 16/3$$